Curvelet applications to asteroseismic data

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In collaboration with
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Summary

- Introduction
- The Curvelet transform
- Applications:
  - Mode identification:
    - Long Runs
    - Short Runs
  - Estimation of mode parameters
- Conclusions

Present situation

- Framework:
  - Asteroseismology (solar-like stars)
  - COROT, MOST, ground-based instrumentation
- Only low-degree modes
- Difficulties to identify oscillations modes
  - Low S/N
- Unknown / low accuracy rotation rate & angle of axis inclination

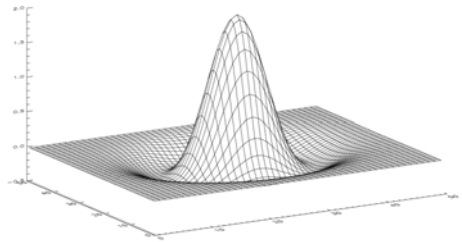
Image processing

- Multi-resolution techniques :
  - Applied in astrophysical image processing
    - Galaxies, planetology,…
  - Restoring an image from noisy data
  - Enhancing patterns embedded in noisy images
- Several processing techniques:
  - Wavelets, Ridgelets, Curvelets
- Application to asteroseismology studies:
  - Echelle diagram
  - Ridge structures of the modes

Toward the Curvelet transform

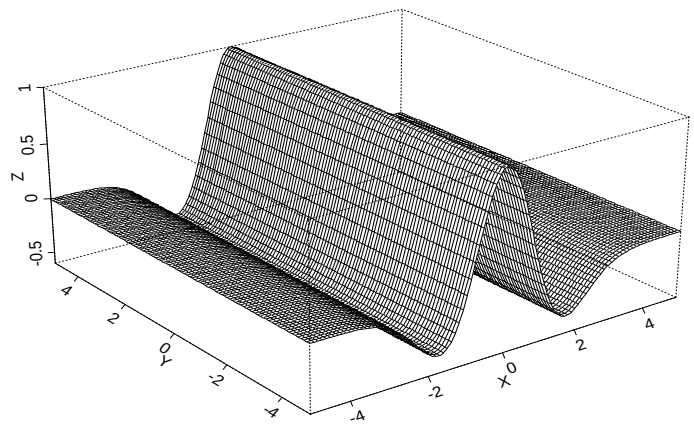
- Wavelets:

$$W(a, b_i, b_j) = \frac{1}{\sqrt{a}} \iint f(x, y)\psi^* \left(\frac{x - b_i}{a}, \frac{y - b_j}{a} \right) dxdy$$



- Ridgelets (Candès, 1998):

$$\frac{1}{\sqrt{a}}\psi\left(\frac{x_1 \cos\theta + x_2 \sin\theta - b}{a}\right)$$



- Parameters:
  - scale *a*, location *b*
  - orientation *θ*

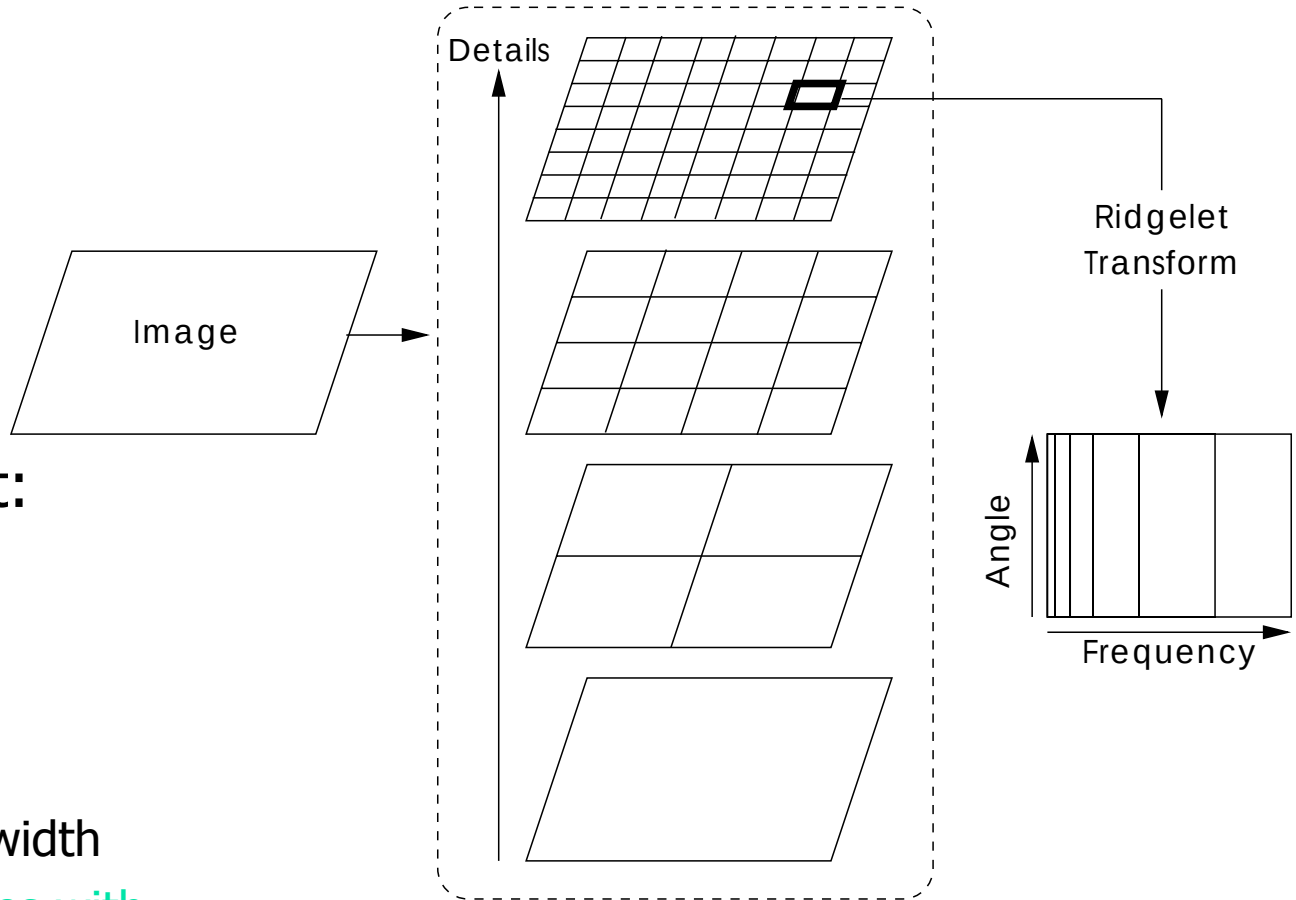
At sufficiently fine scale, a curved edge is almost straight

→ *Ridgelets in a localized manner, at sufficiently fine scales*

➔ **the curvelet transform**

The curvelet transform

- Curvelets (Candès & Donoho,1999; Starck, Donoho & Candès, 2003):



- Edges at different:
  - scale,
  - location,
  - orientation,
  - size,
  - variable length & width
  - anisotropy increases with decreasing scale

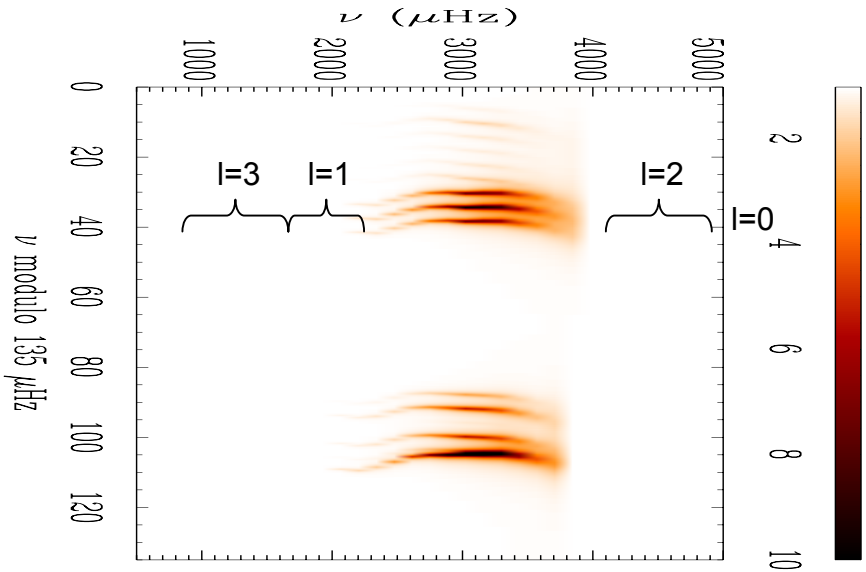
In practice

- FFT or autocorrelation of the seismic spectrum
  → mean large separation
  → echelle diagram
- Curvelet filtering
- Mode identification (who's who? l=0,1,2,…)
- Use of the filtered spectrum?
  - Guesses for peak bagging
  - Estimation of mode parameters:
    - Rotational splitting, inclination angle, frequencies

Models

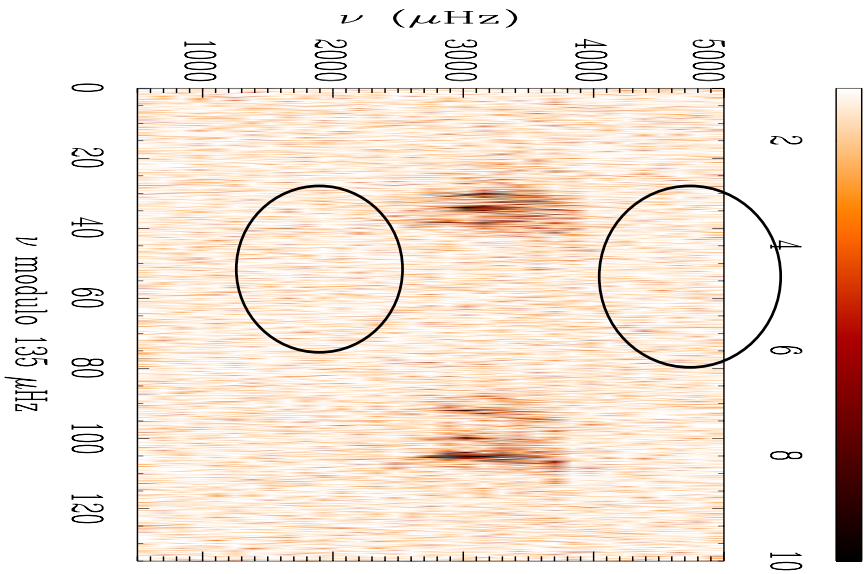
- Sun-like case
- Solar-like signal embedded in χ^2 with 2 d.o.f. distribution noise
  - l=0,1,2,3 and n=12-25 ⇔ 2080-3780 μHz
- Set of:
  - S/N
  - Inclination angles
  - Rotation rates
  - Long and short runs
- Focus on a fast and tilted Sun:
  - i = 50°
  - $\Omega = 10.\ \Omega_\odot$

Mode identification: long runs



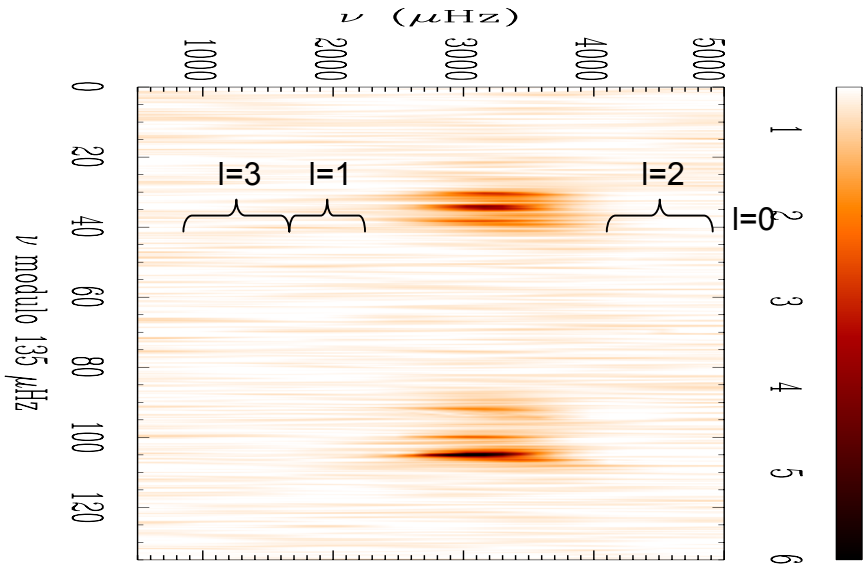
Theoretical echelle diagram

S/N = 15



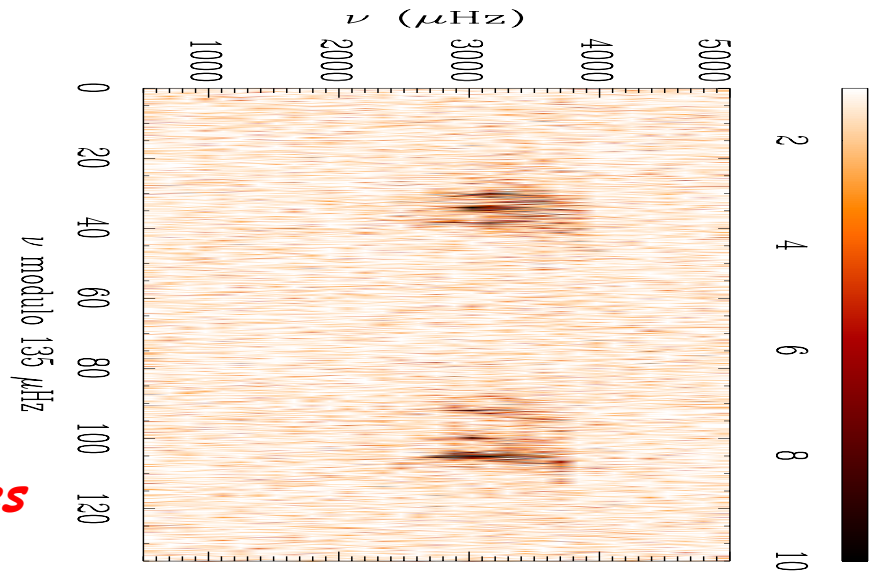
Raw echelle diagram

Mode identification: long runs



Filtered echelle diagram

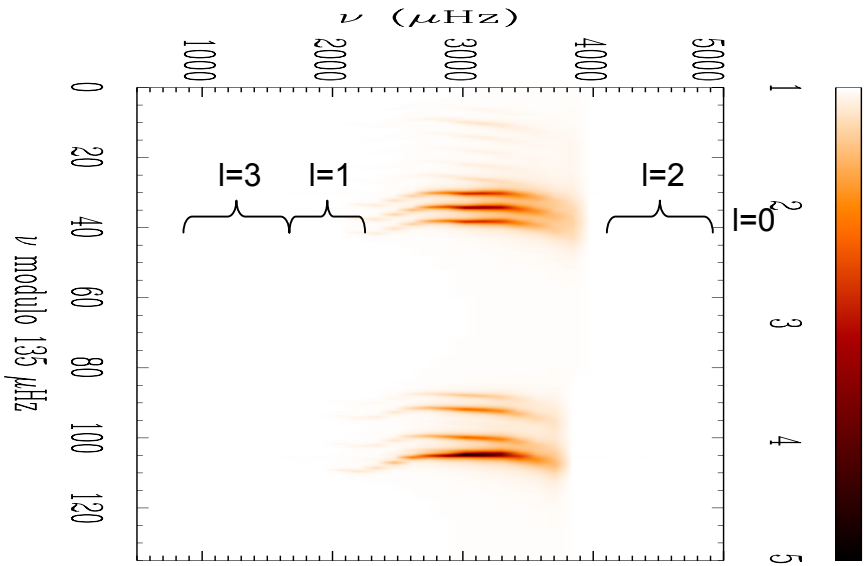
S/N = 15



Raw echelle diagram

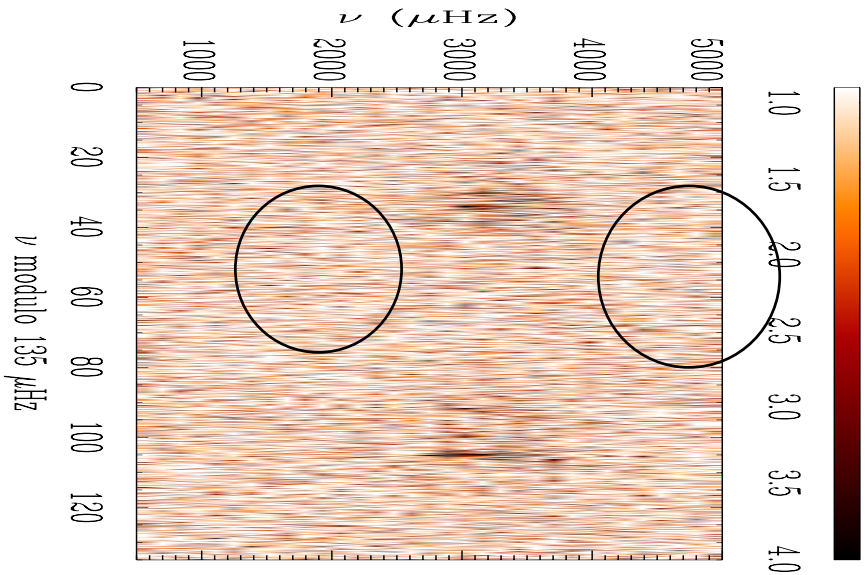
Denoising → Better contrast
→ Identification of the ridges improved on the diagram

Mode identification: long runs



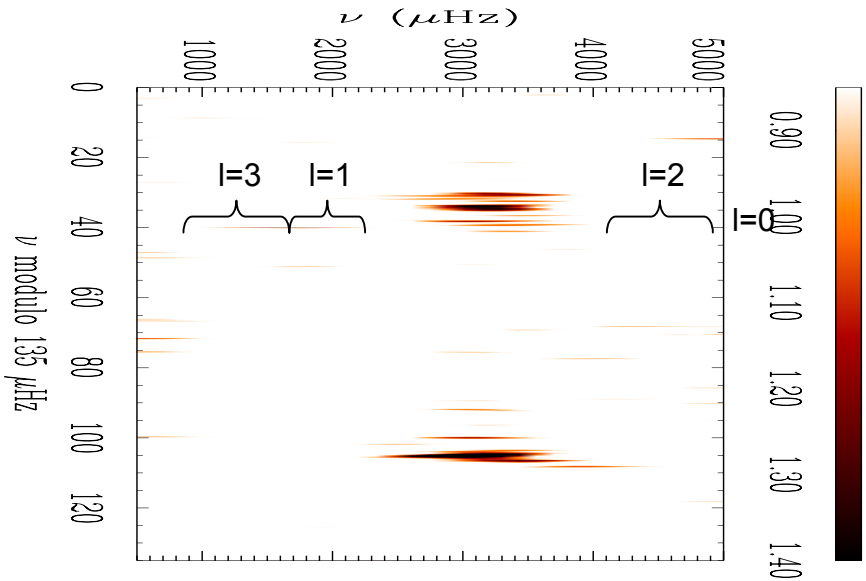
Theoretical echelle diagram

S/N = 5



Raw echelle diagram

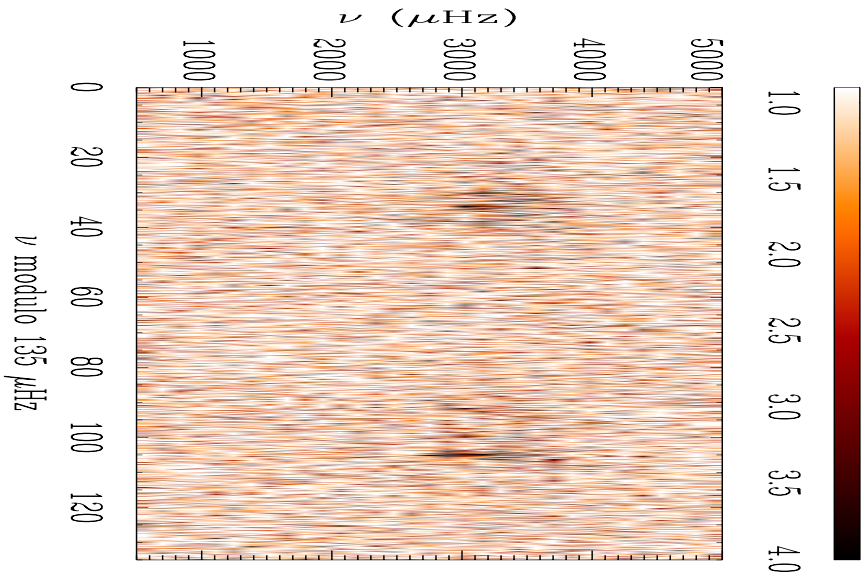
Mode identification: long runs



Filtered echelle diagram

Better enhancement for lower SNR

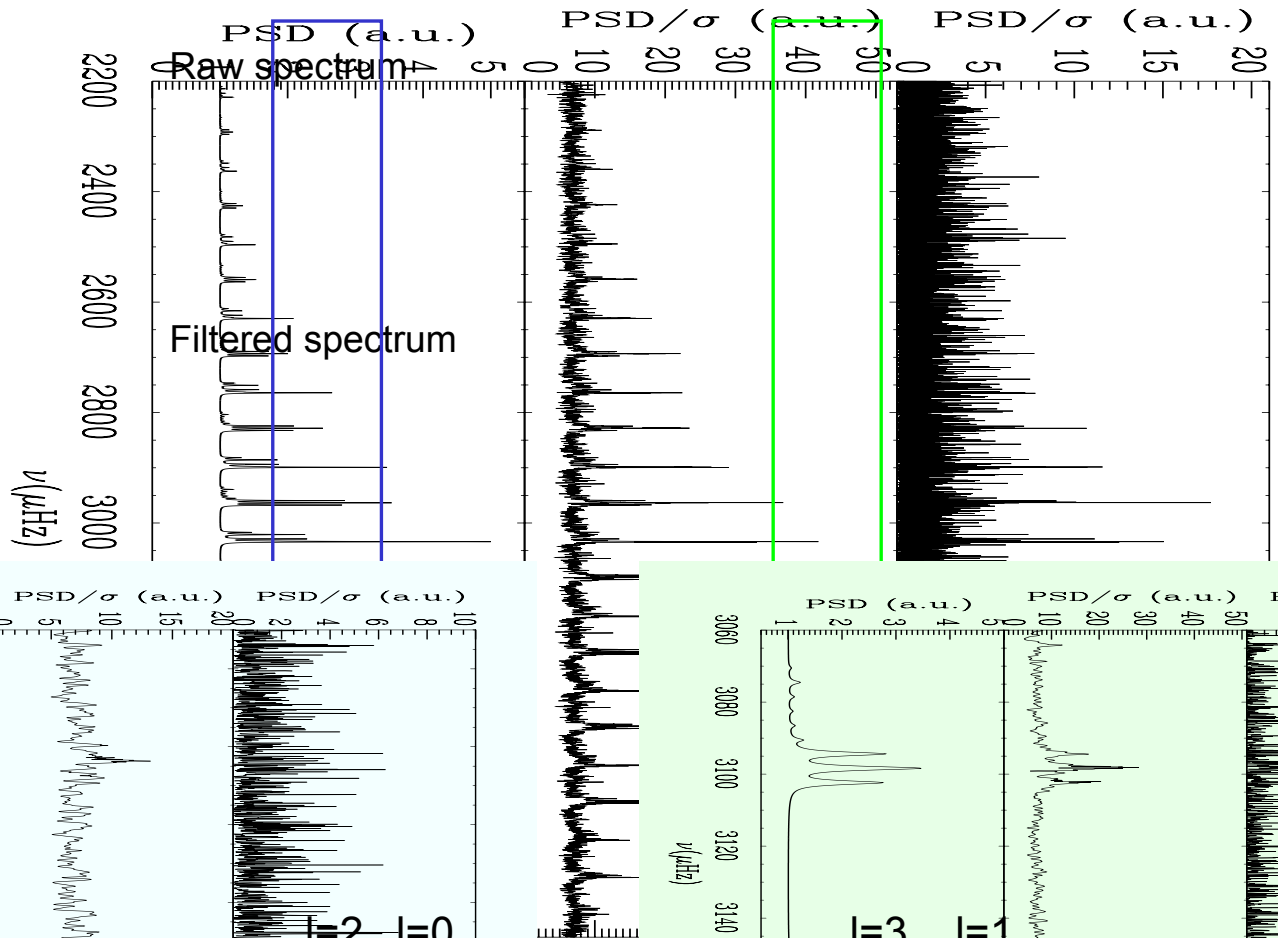
S/N = 5

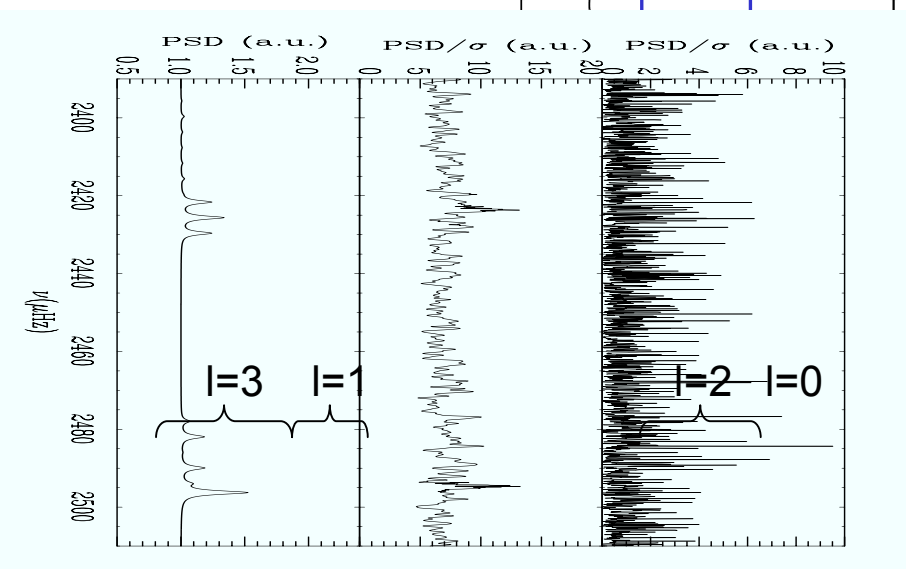


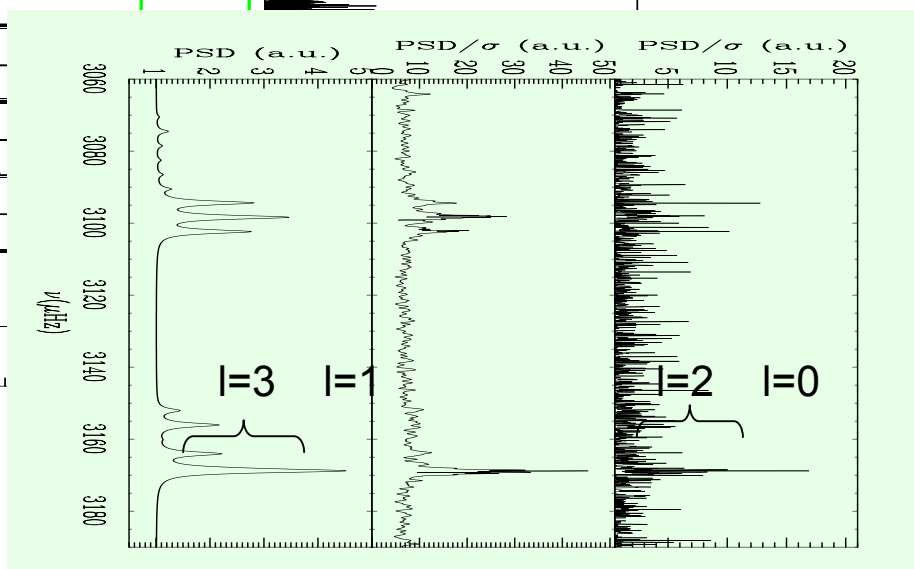
Raw echelle diagram
(Useless in this case)

Mode identification: long runs

S/N = 5





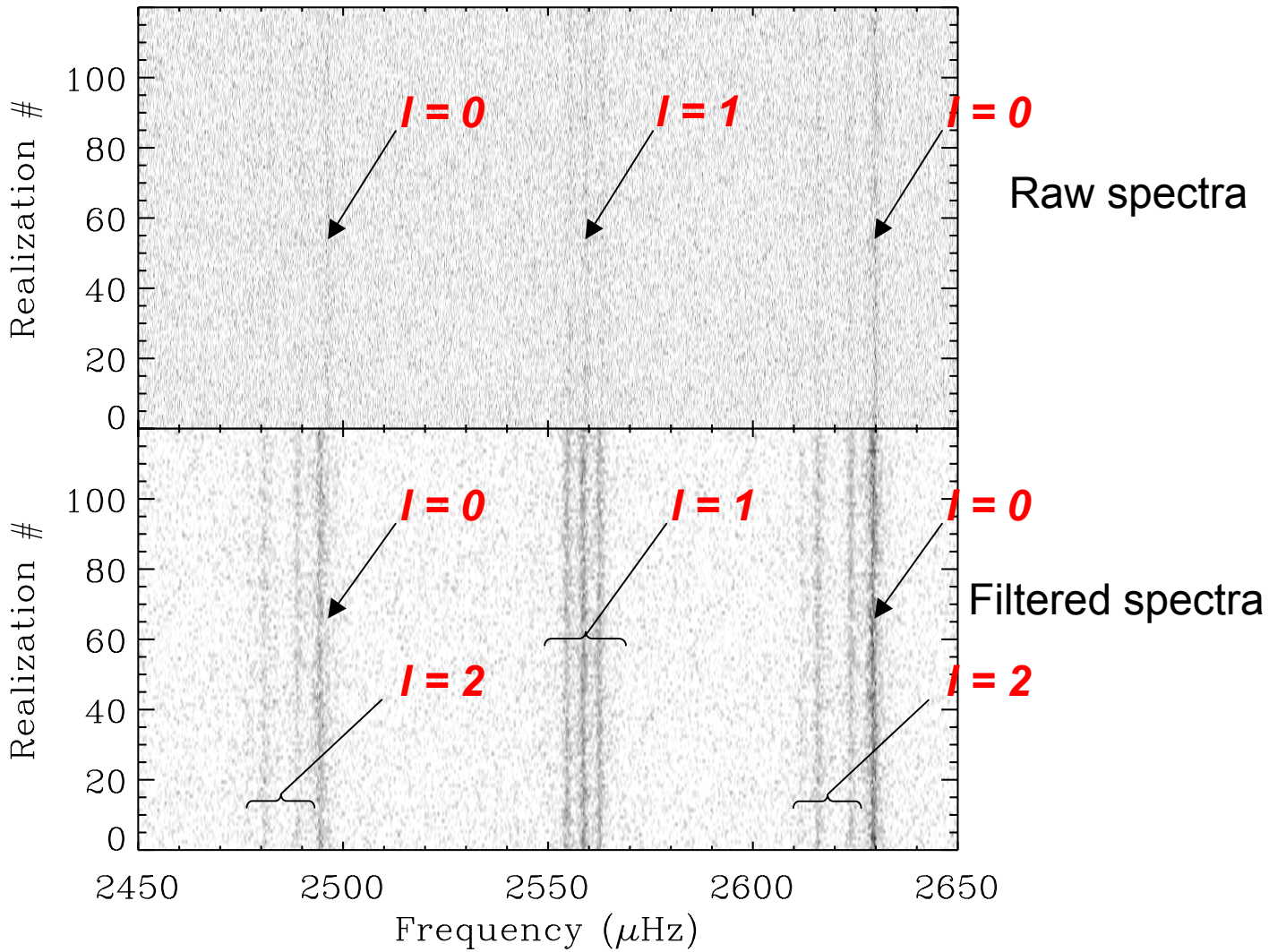


Mode identification: long runs

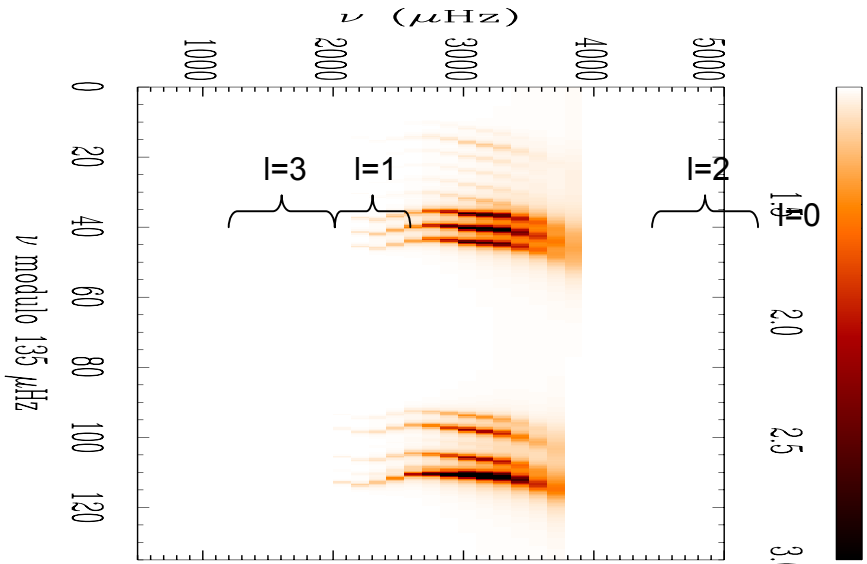
S/N = 5

Monte Carlo realizations

Success rate increased after filtering

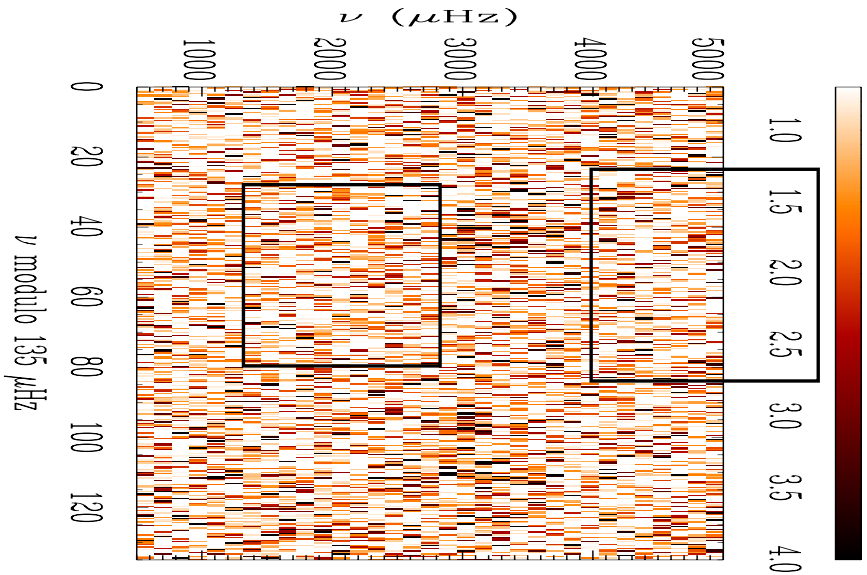


Mode identification: short runs



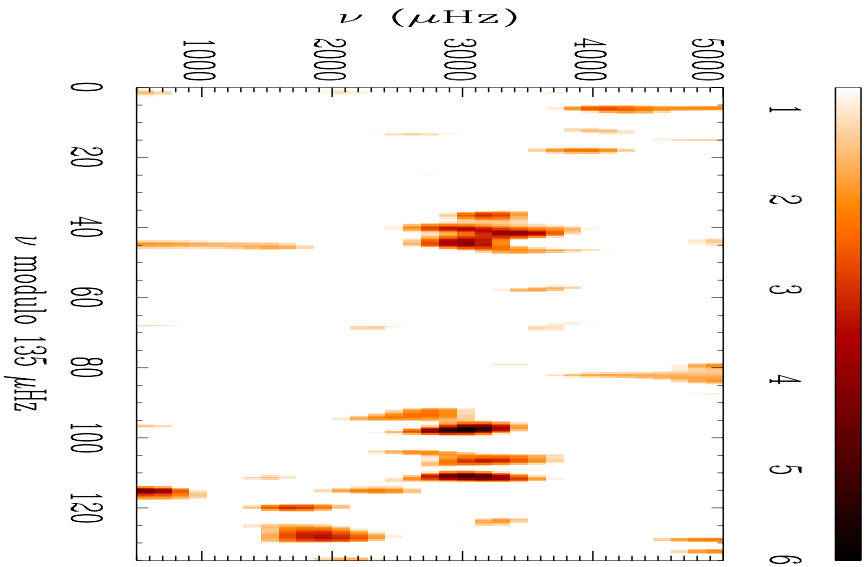
Theoretical echelle diagram

S/N = 5



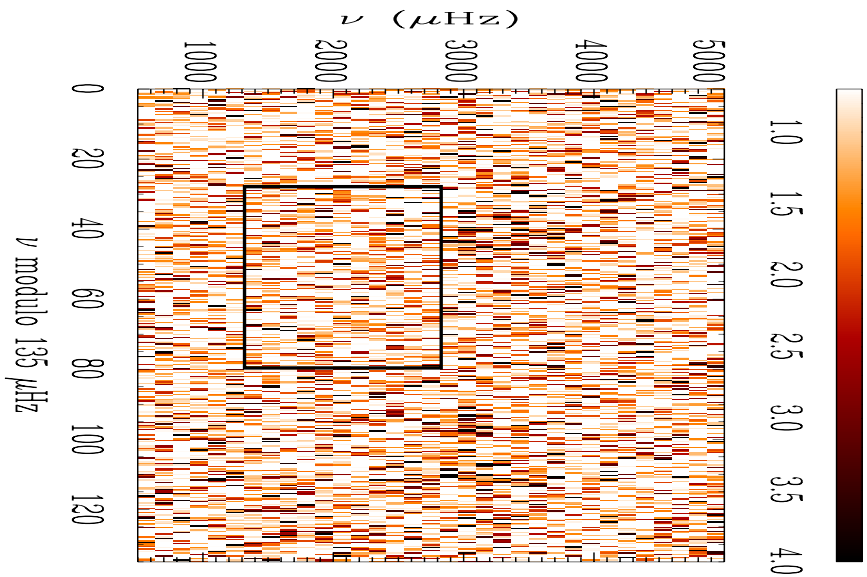
Raw echelle diagram

Mode identification: short runs



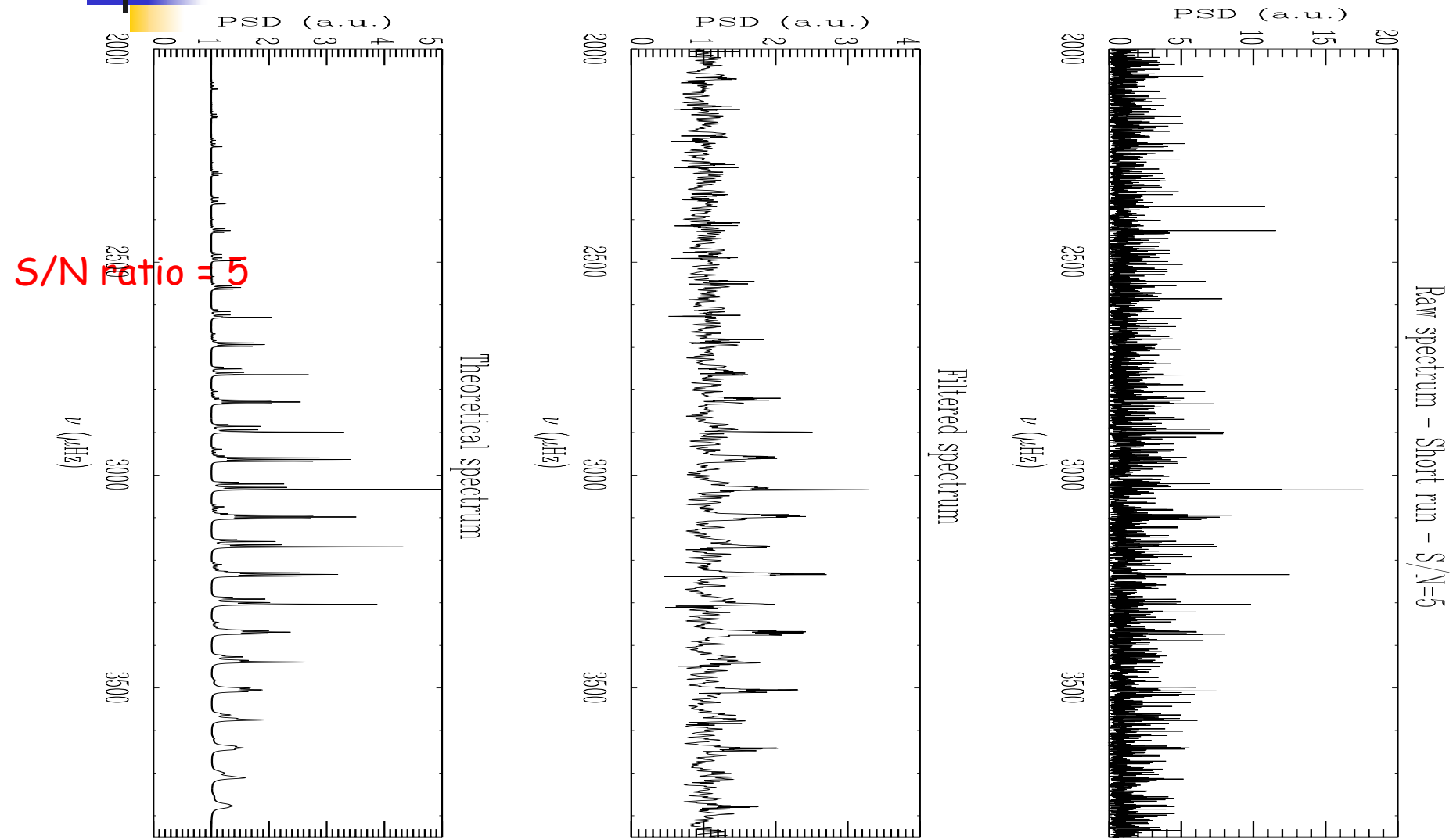
Filtered echelle diagram

S/N = 5



Raw echelle diagram

Mode identification: short runs



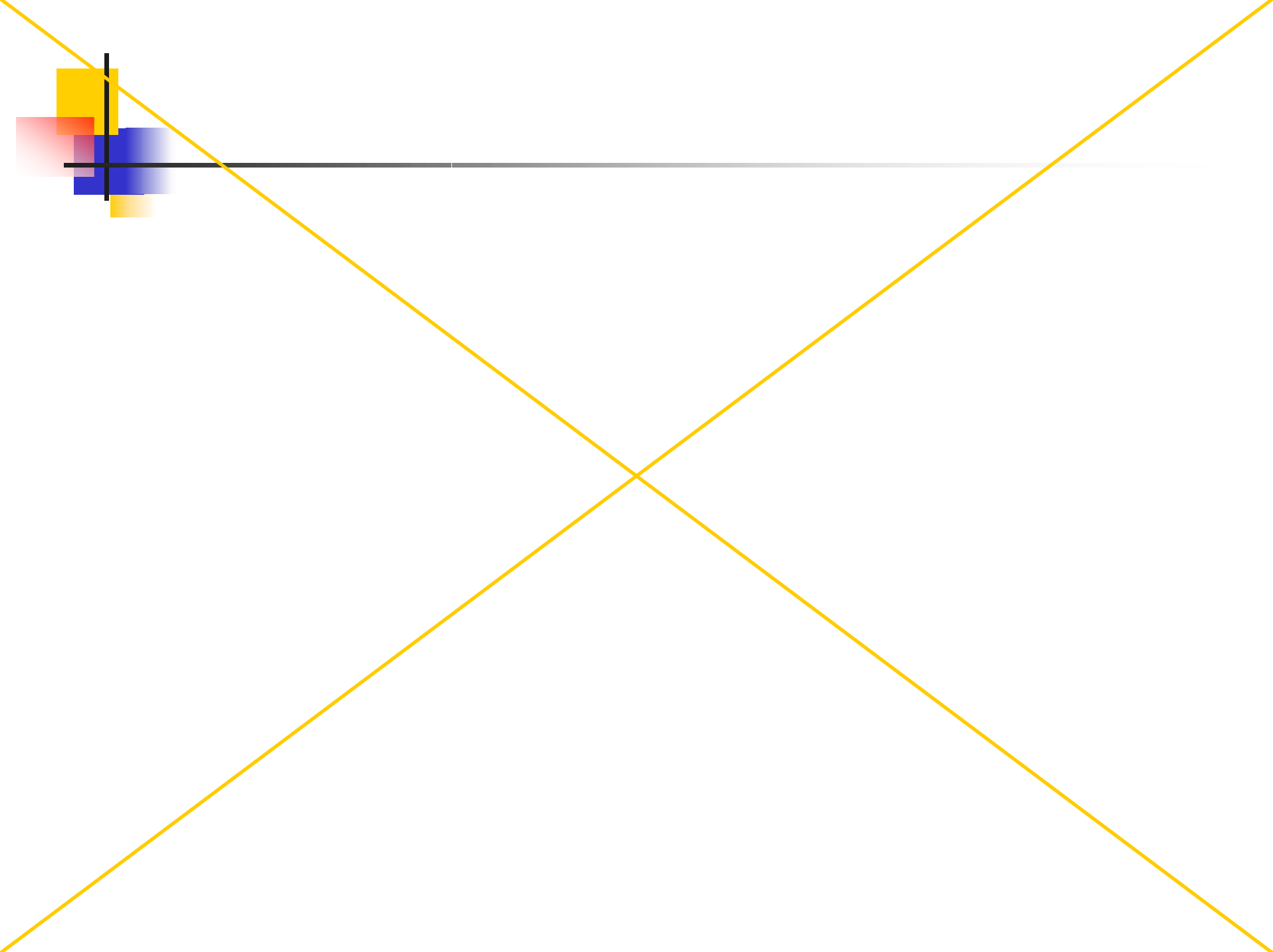
Mode-parameter extraction

- Low S/N:
  - more modes can be seen
  - possible extraction of some mode parameters:
    - frequencies, amplitude ratio
      ➔ estimation of inclination angle, rotational splitting
    - used as guesses to fit the raw spectrum

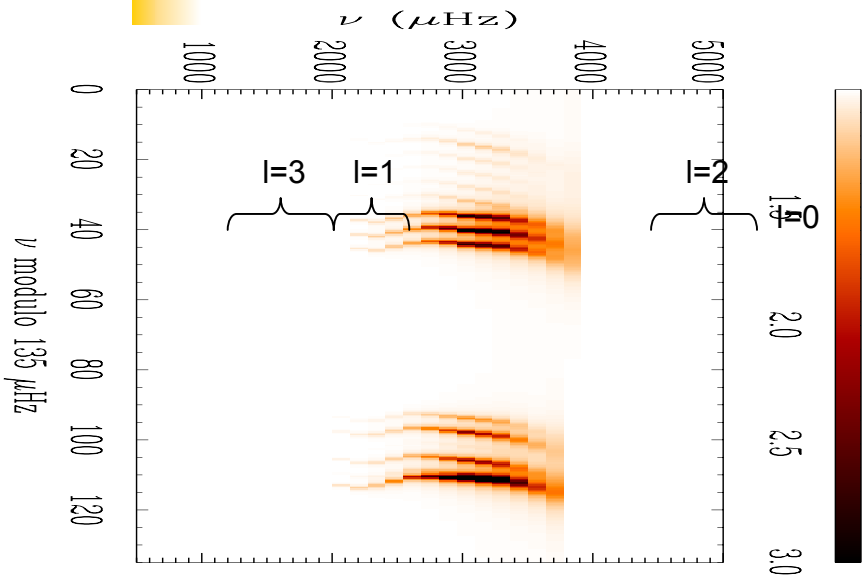
- Aliases at $(k.\Delta\nu)$ at low and high frequency
  - amplified by the sudden cut of our simulated signal
  - possible to identify and to discriminate them

Perspectives - Conclusions

- Promising method to improve mode identification
- Efficiency depends on S/N and observation time
  - Adapted to short runs
- Estimation of inclination angle & rotational splitting
- Easy way to obtain guesses for the fit of raw series
- Aliases at low and high frequency
  - Easily identified
- *Paper coming soon: Lambert, Pires, Ballot, García & Starck, A&A*

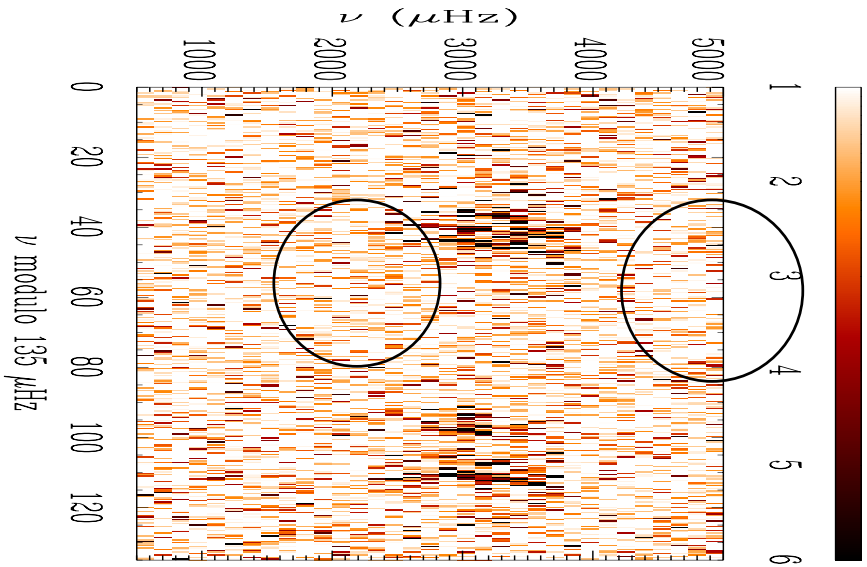


Mode identification: short runs



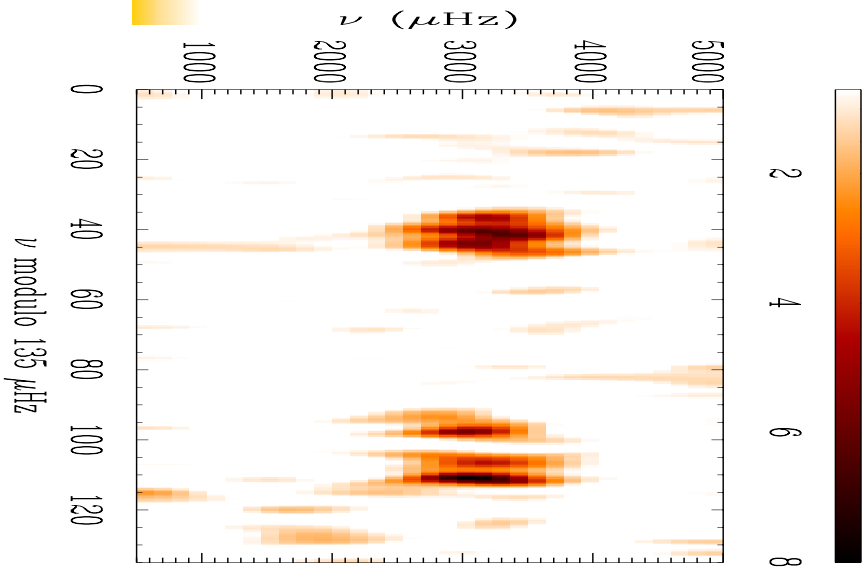
Filtered echelle diagram

S/N = 15



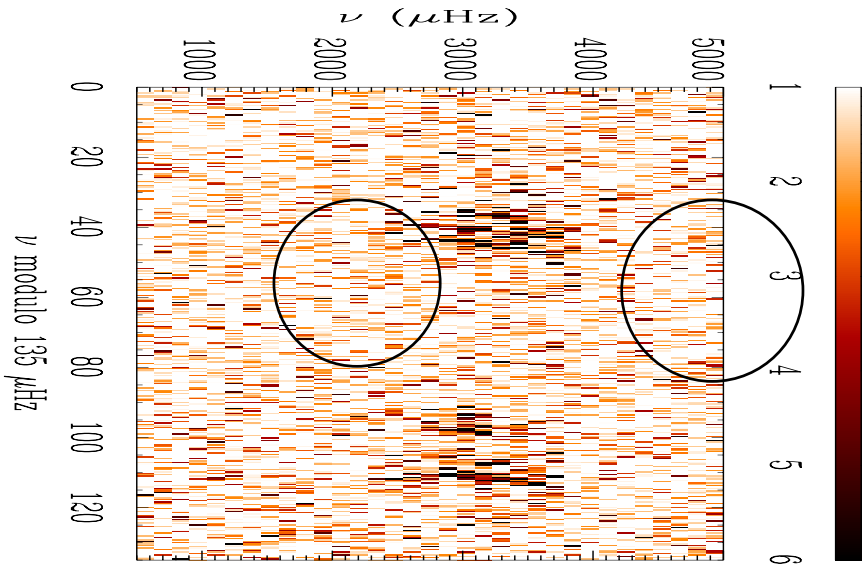
Raw echelle diagram

Mode identification: short runs



Filtered echelle diagram

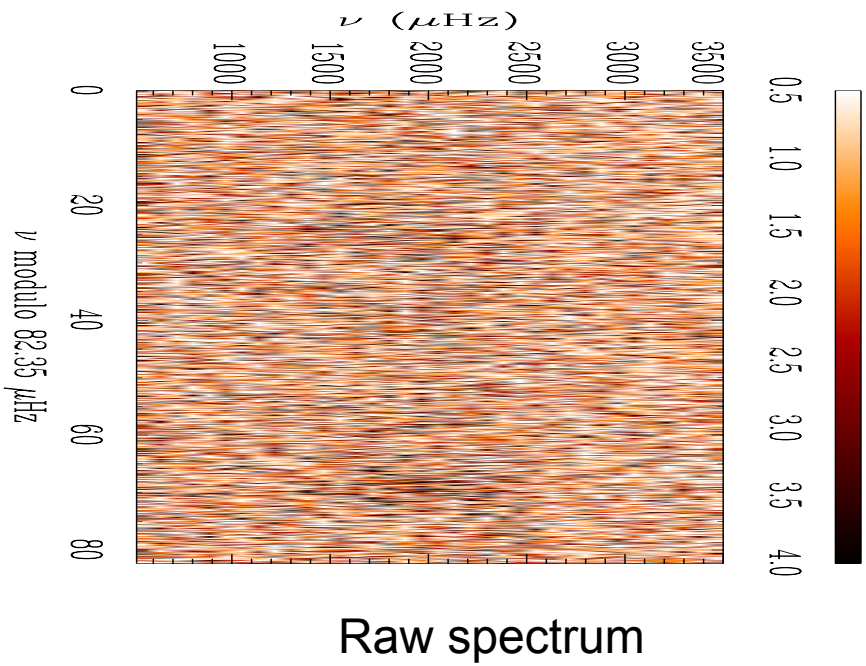
S/N = 15



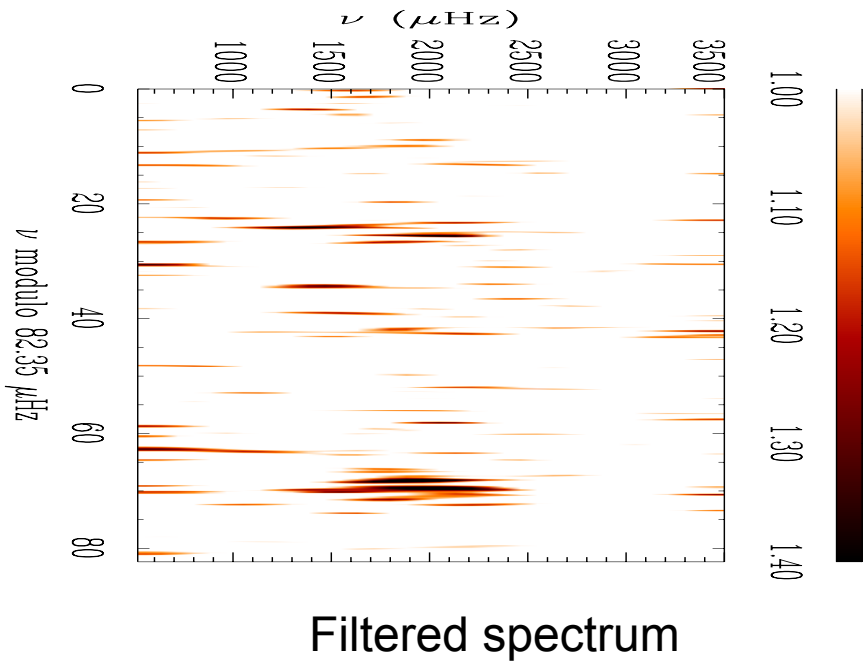
Raw echelle diagram

Mode identification: *hh4*

HH4

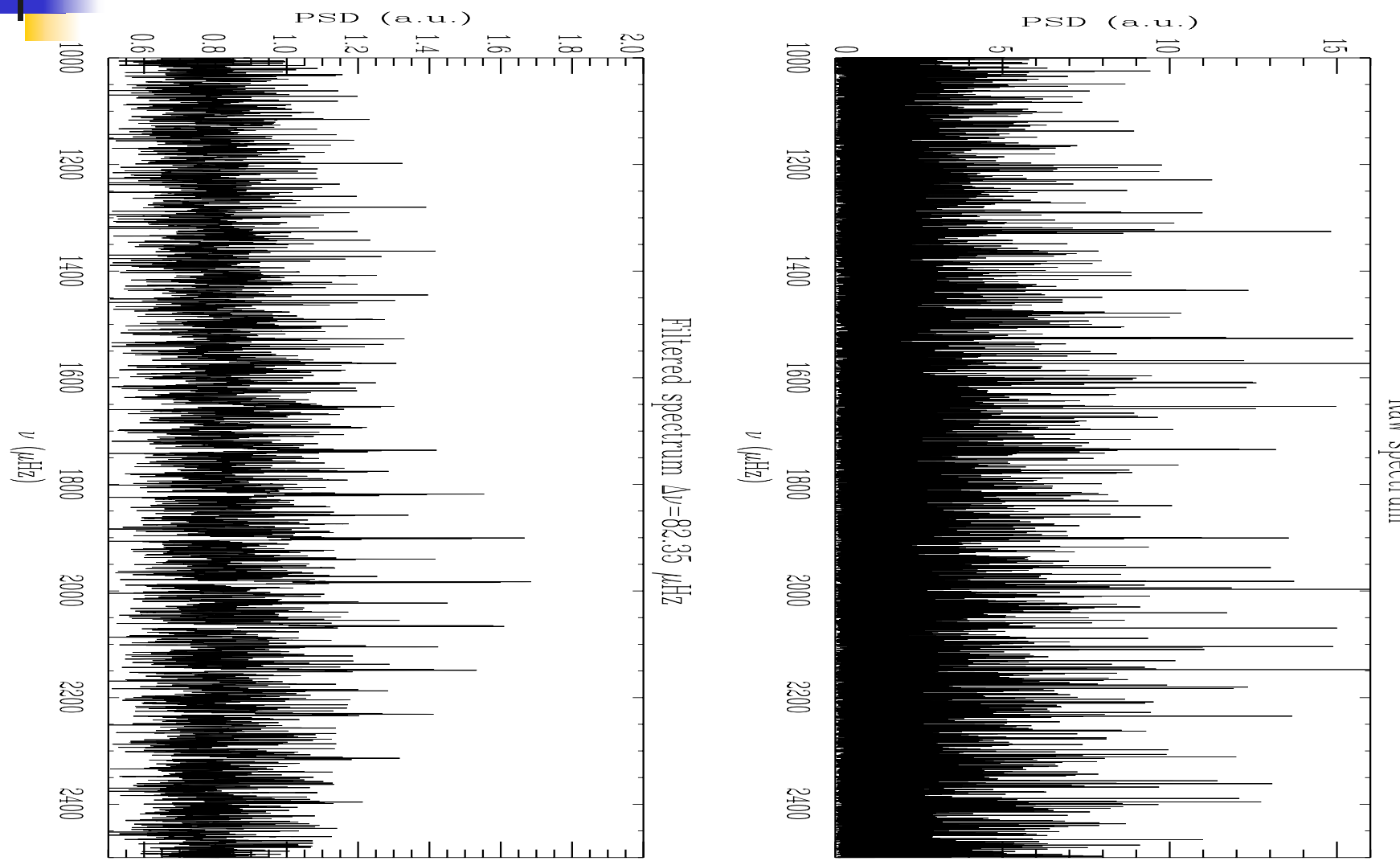


Raw spectrum

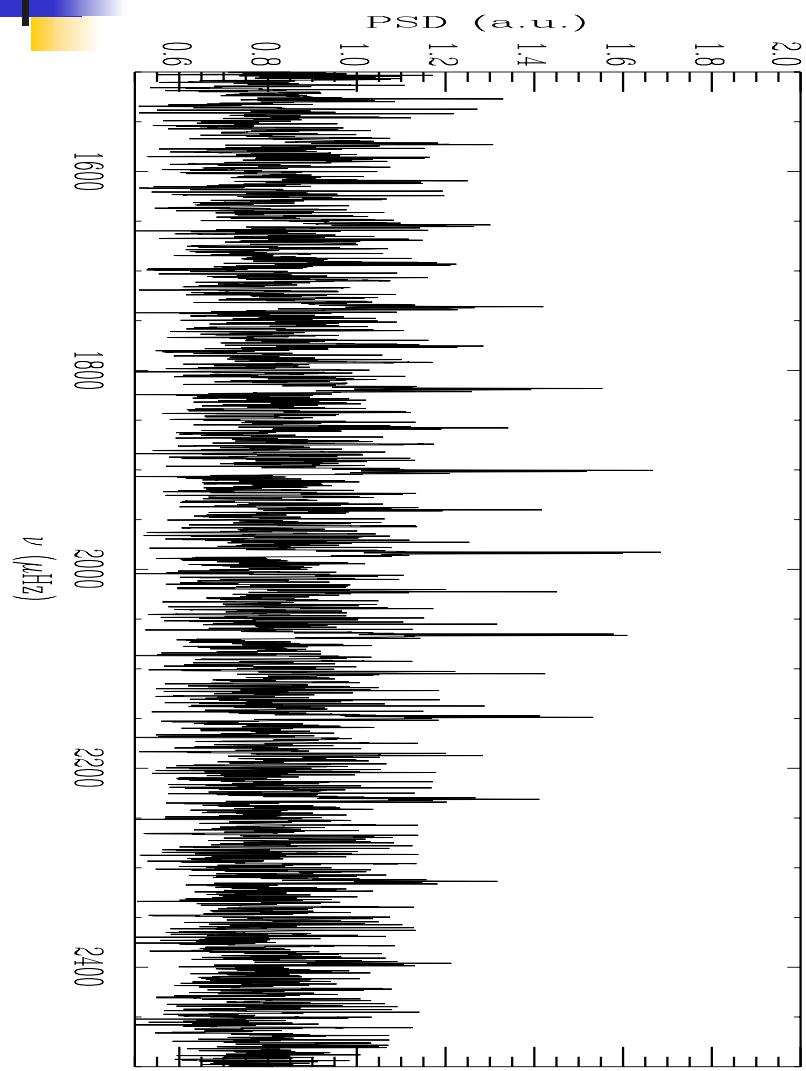


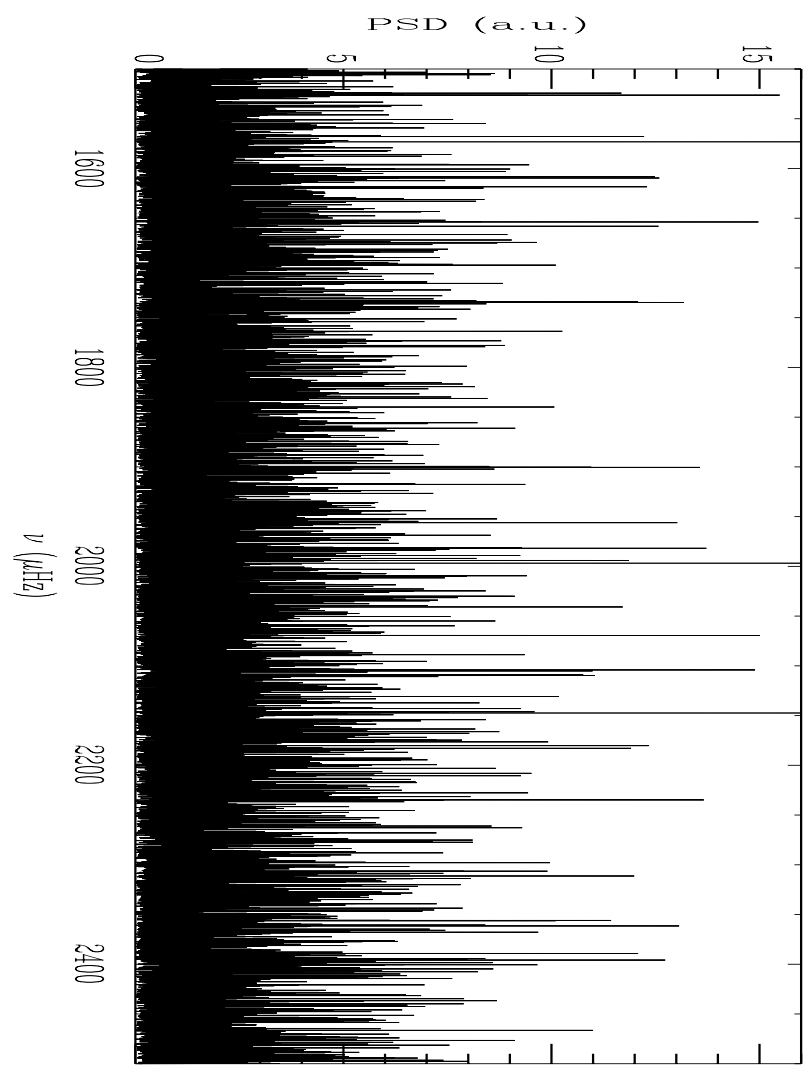
Filtered spectrum

Mode identification: *hh4*

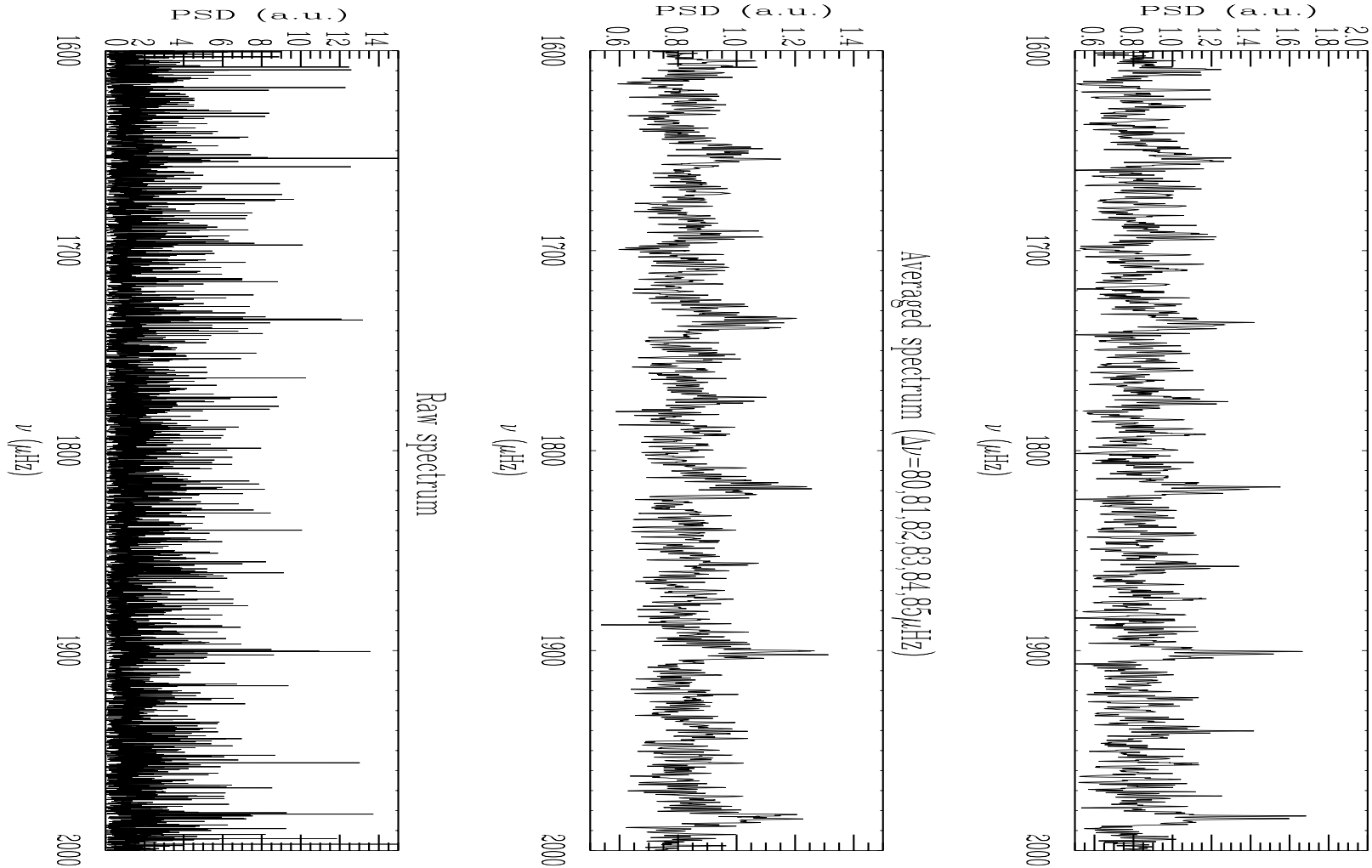


Mode identification: *hh4*

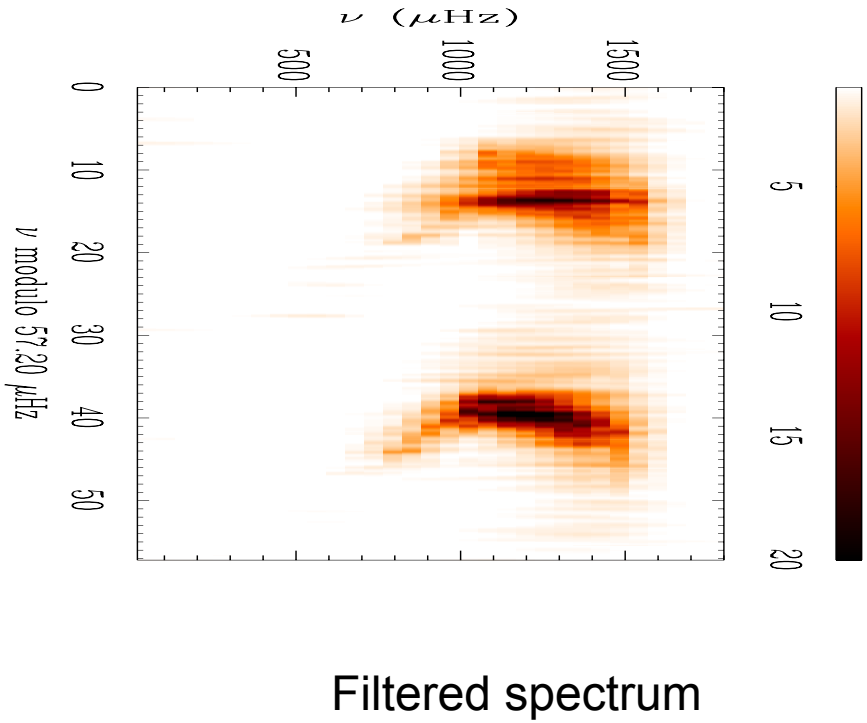




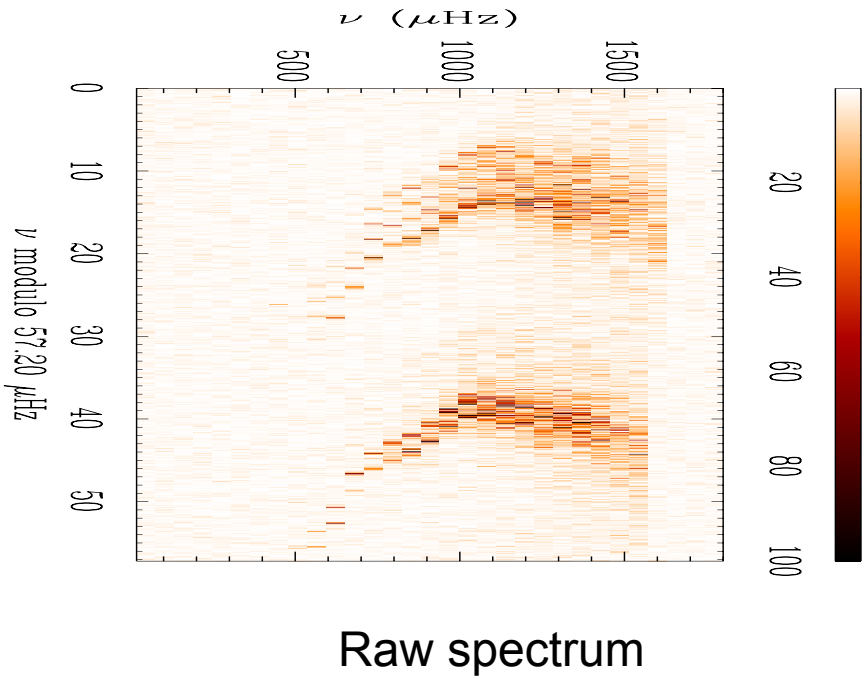
Mode identification: *hh4*



Mode identification: HD43318

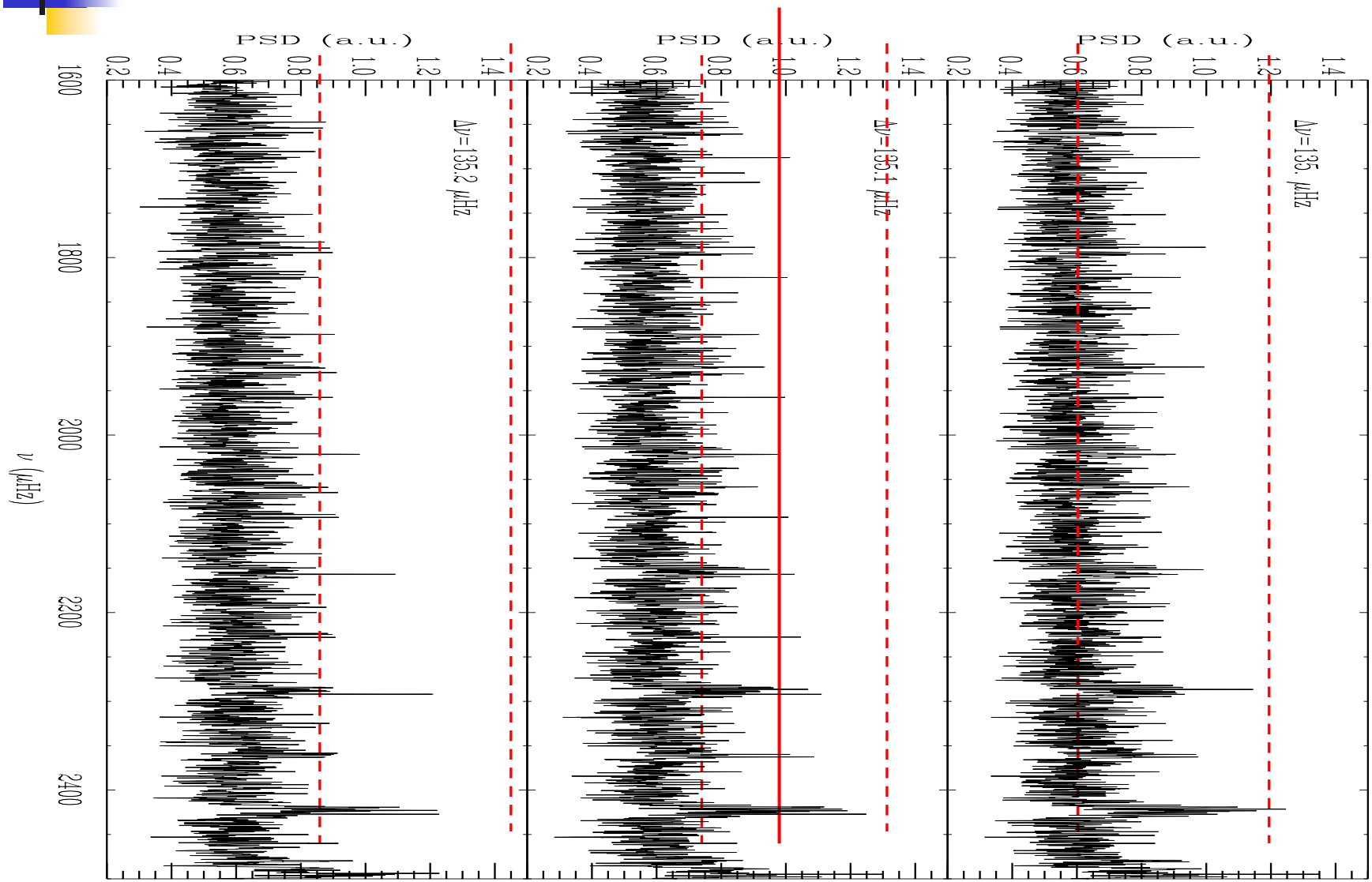


Filtered spectrum



Raw spectrum

Artefacts: *sensitivity to Δν*



Artefacts: *sensitivity to Δv*