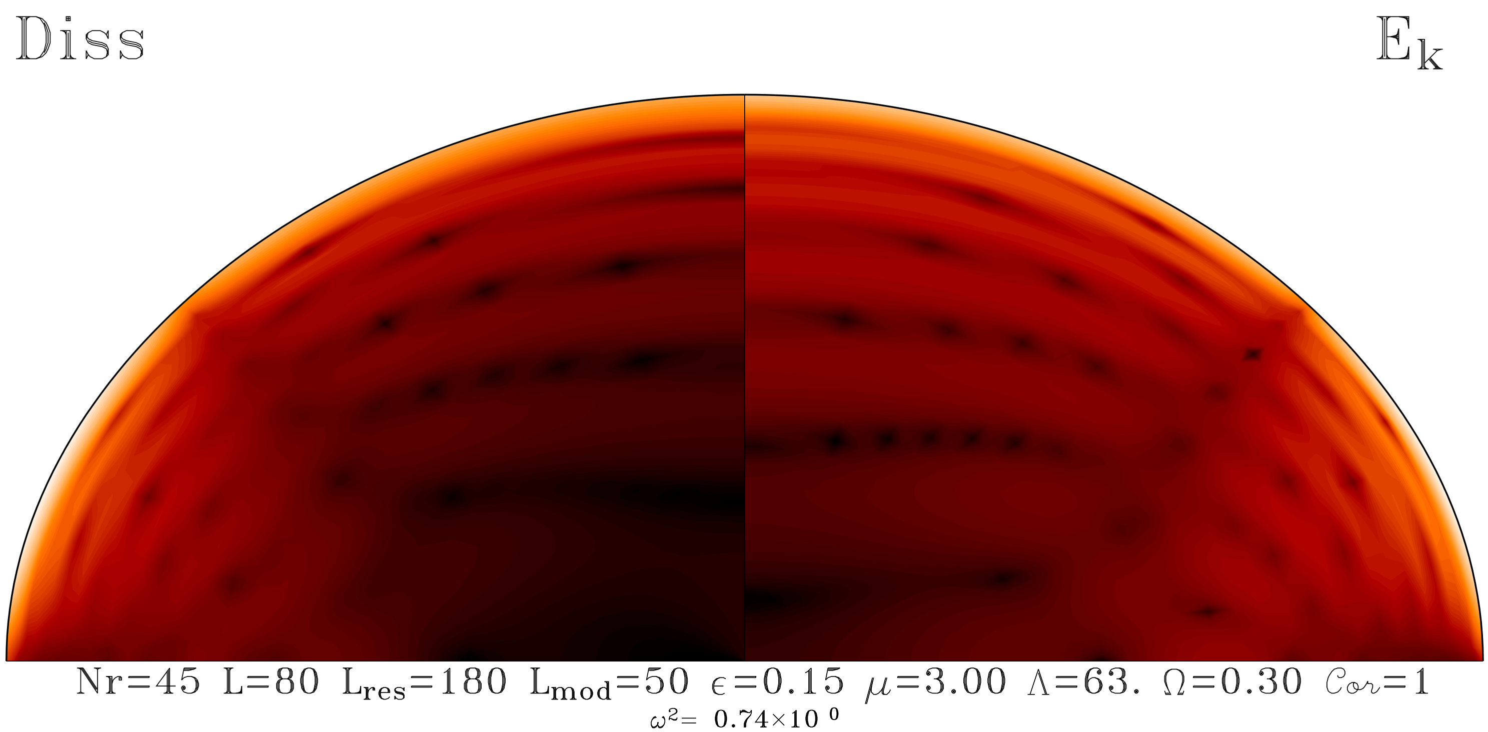
Oscillations of rapidly rotating stars



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Introduction

- ✘ Accurate pulsation frequency measurements are expected from Corot
- ✘ Case of rapidly rotating stars :
  - for example : δ Scuti and γ Doradus stars
  - need for accurate predictions
  - theoretical challenge

Two approaches :

- Perturbative
- Numerical

Other people's work

Perturbative methods (valid for small rotation rates)

✘ *Description* :
the equilibrium structure and the oscillation modes are the sum of two parts :
a non-rotating solution + a deviation

✘ *Some references* :

- 2^{nd} order methods :
  - Saio (1981)
  - Gough and Thompson (1990)
  - Dziembowski and Goode (1992)
- 3^{rd} order methods :
  - Soufi, Goupil and Dziembowski (1998)
  - Karami et al. (2005)

Numerical methods (for any rotation rates)

✘ *Description* :
direct numerical computation of 2D equilibrium structure and oscillation modes

✘ *Some references* :

- Clement (1986)
- Yoshida and Eriguchi (2001)

Our work

- Direct numerical method
- First time results from a direct 2D numerical method have been compared with results from the perturbative approach
- Equilibrium model : uniformly rotating polytropic model of a star

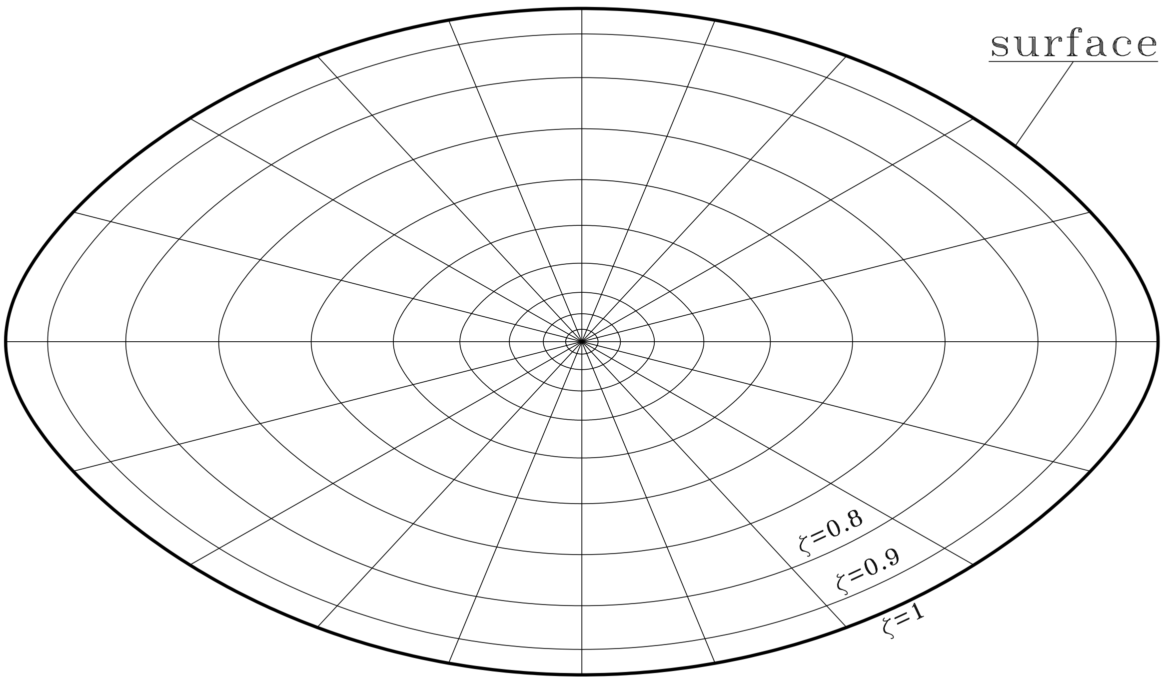
$$\begin{aligned} P_o &= K\rho_o^\gamma \\ 0 &= -\vec{\nabla} P_o - \rho_o \vec{\nabla}\left(\phi_o + \Omega^2 s^2\right) \\ \Delta\phi_o &= 4\pi G\rho_o \end{aligned}$$

- Linearised adiabatic oscillations :

$$\begin{aligned} i\omega\rho &= -\vec{\nabla}\cdot(\rho_o\vec{v}) \\ i\omega\rho_o\vec{v} &= -\vec{\nabla} p + \rho\vec{g}_o - \rho_o\vec{\nabla}\Phi - 2\rho_o\vec{\Omega}\times\vec{v} \\ i\omega\left(p - c_o^2\rho\right) &= \frac{\rho_o c_o^2 N_o^2}{\|\vec{g_o}\|^2}\vec{v}\cdot\vec{g}_o \\ \Delta\Phi &= 4\pi G\rho \end{aligned}$$

Ingredients to obtaining accurate frequencies

1. Use of adapted coordinate system (ζ, θ, ϕ) [Bonazzola et al., 1998]
2. Use of spectral methods in both directions
3. Choice of well-behaved quantities as unknowns
4. Use of Arnoldi-Chebyshev algorithm



Tests and Accuracy of the method

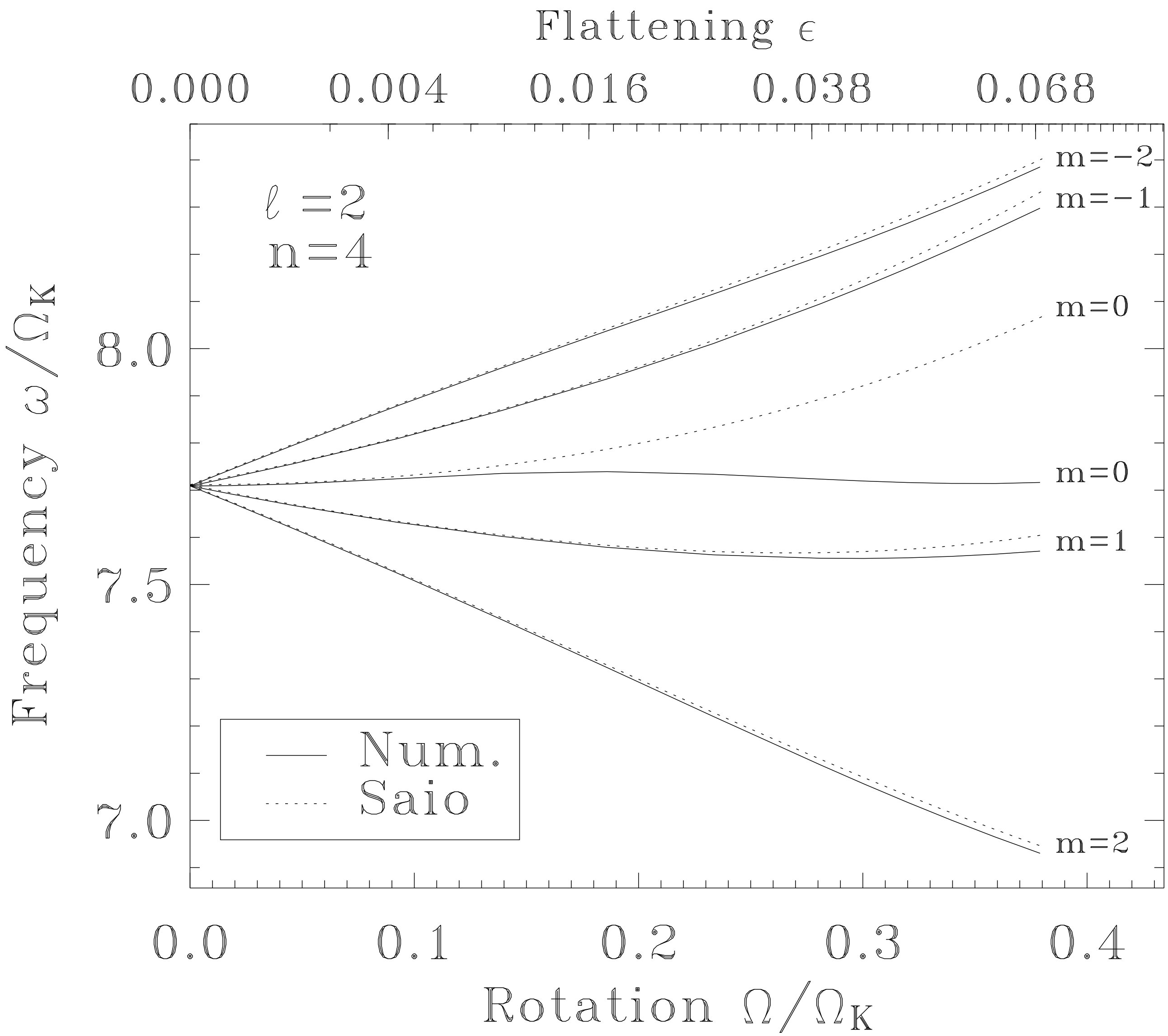
Tests

- Comparison with Christensen-Dalsgaard and Mullan (1994) in the non-rotating case : $\Delta\omega/\omega \sim 10^{-7}$
- Comparison with Lignières (2003, CW5) : $\Delta\omega/\omega \sim 10^{-6}$
- Comparison with Saio for small rotation rates

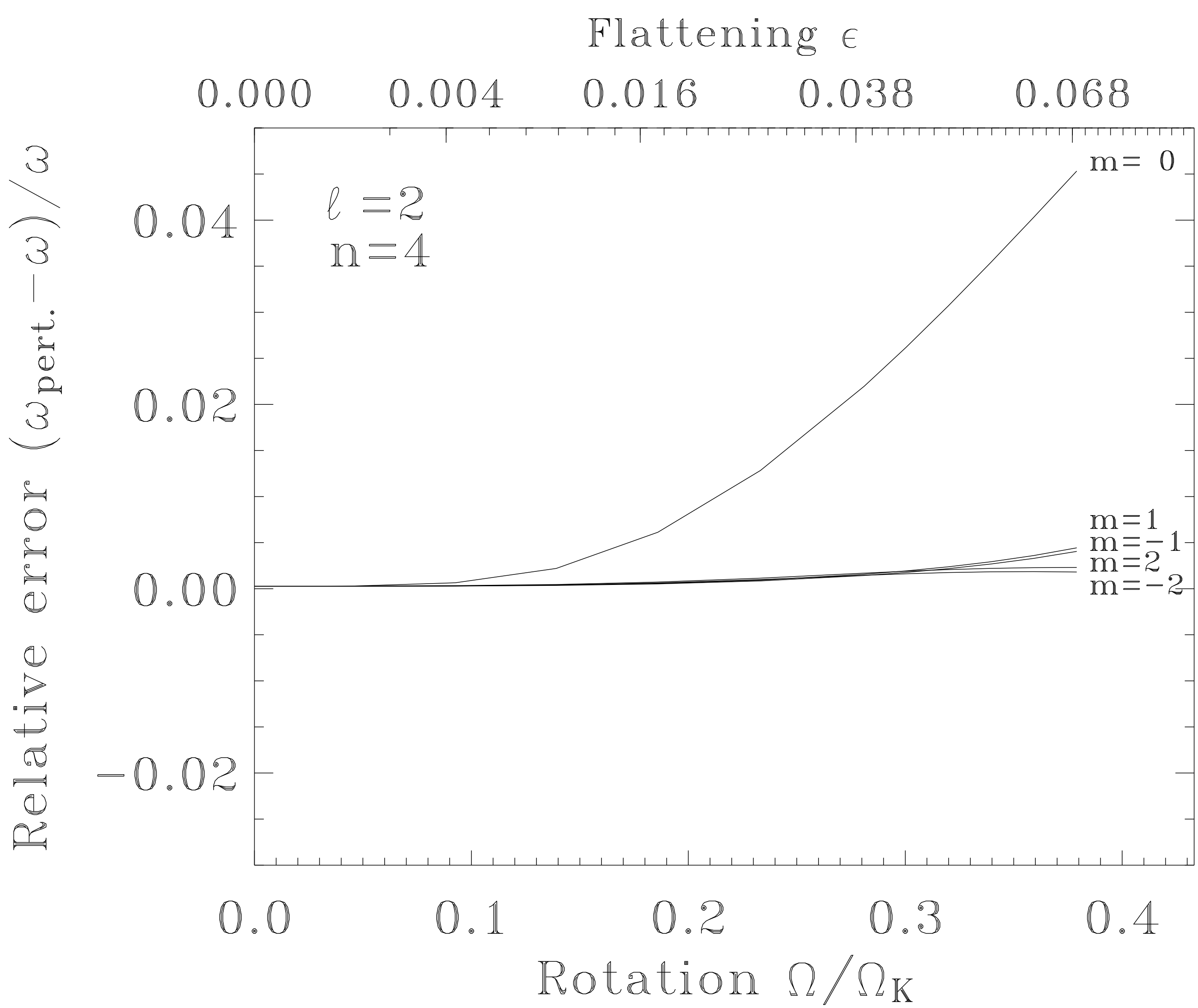
Accuracy

- High numerical stability of frequencies : $\Delta\omega/\omega \sim 10^{-6}$

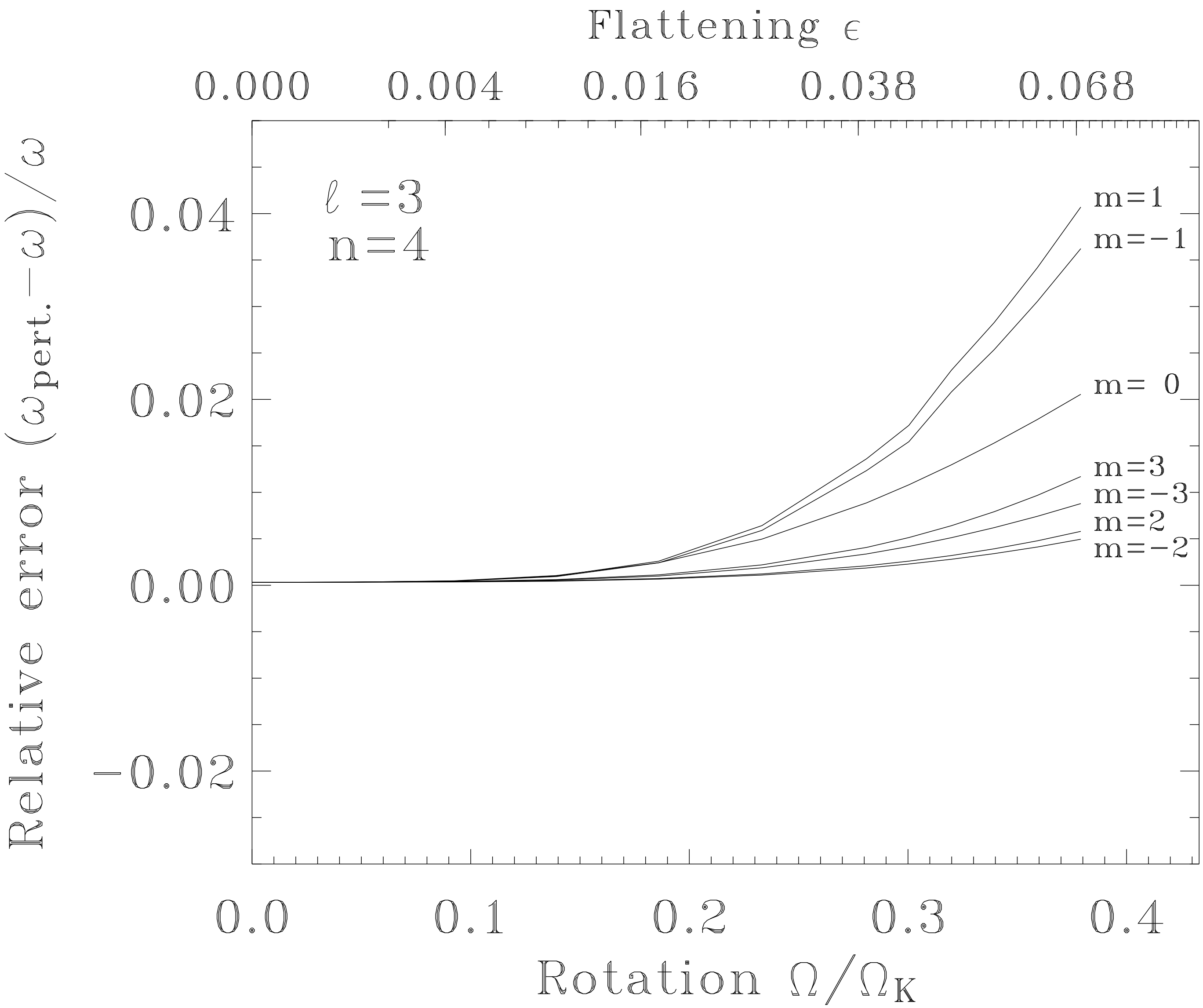
Comparison with 2nd order perturbative method



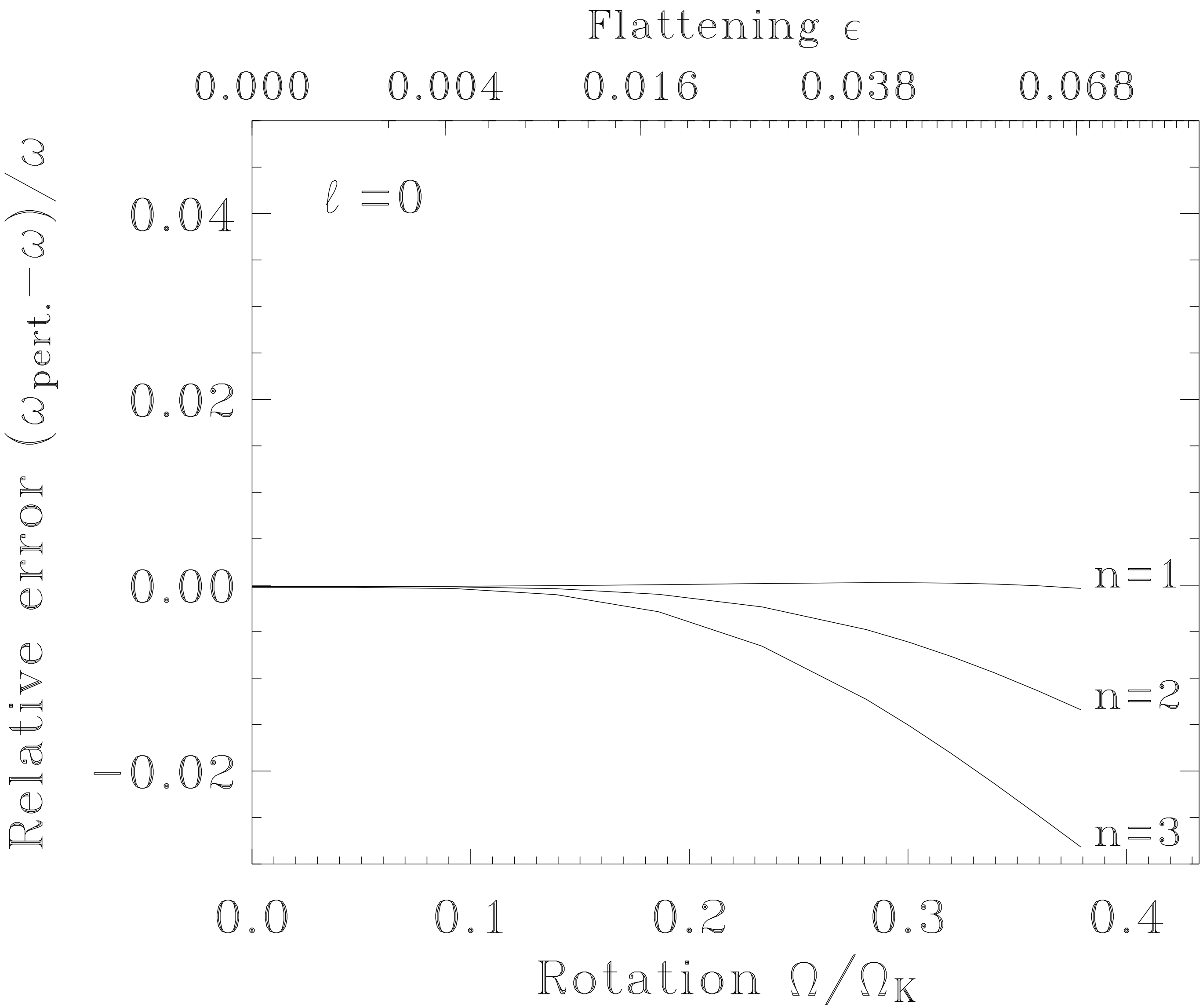
Comparison with 2^{nd} order perturbative method (2)



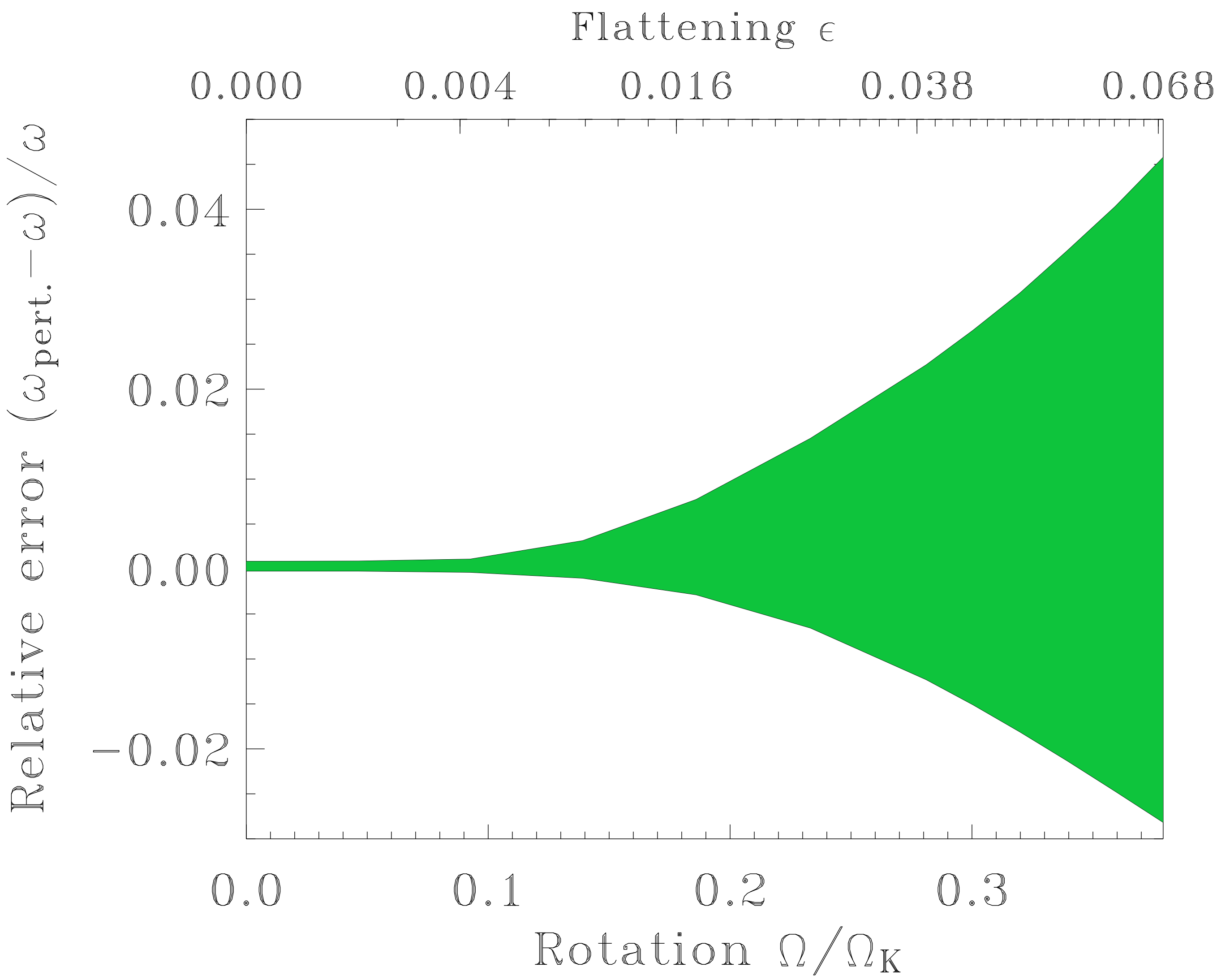
Comparison with 2^{nd} order perturbative method (3)



Comparison with 2^{nd} order perturbative method (4)



Comparison with 2^{nd} order perturbative method (5)



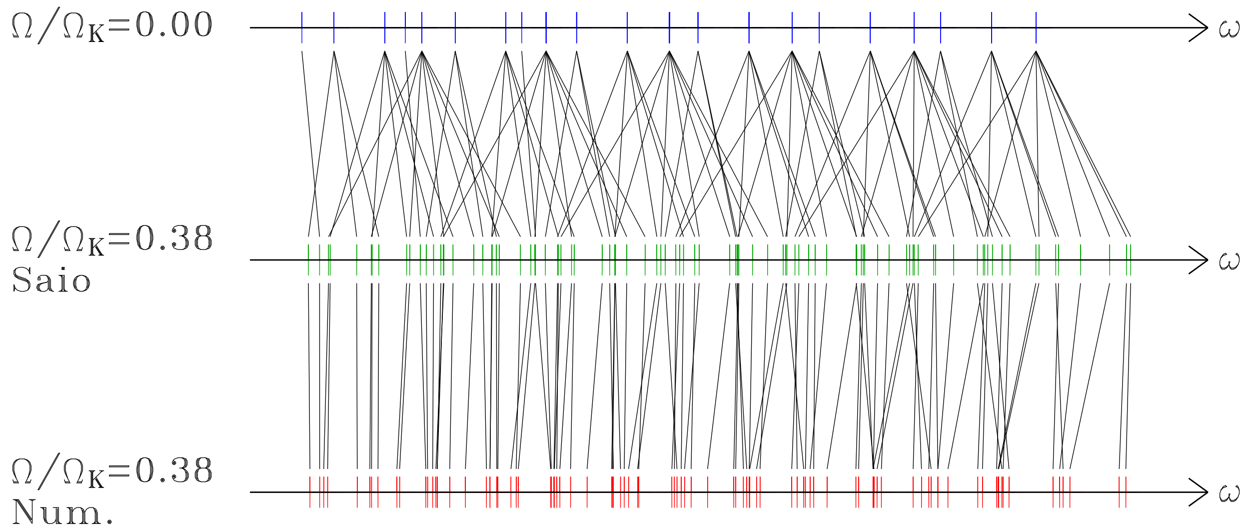
Modes investigated : $\begin{cases} \ell = 0, & n = 1-3 \\ \ell = 1,2,3, & n = 1-6 \end{cases}$

Application to a δ Scuti star

Adopted parameters

- $v \sin i = 170\ km.s^{-1}$
- $i = 90°$
- $M = 2\ M_{\odot}$
- $R_{pol} = 1.75\ R_{\odot}$

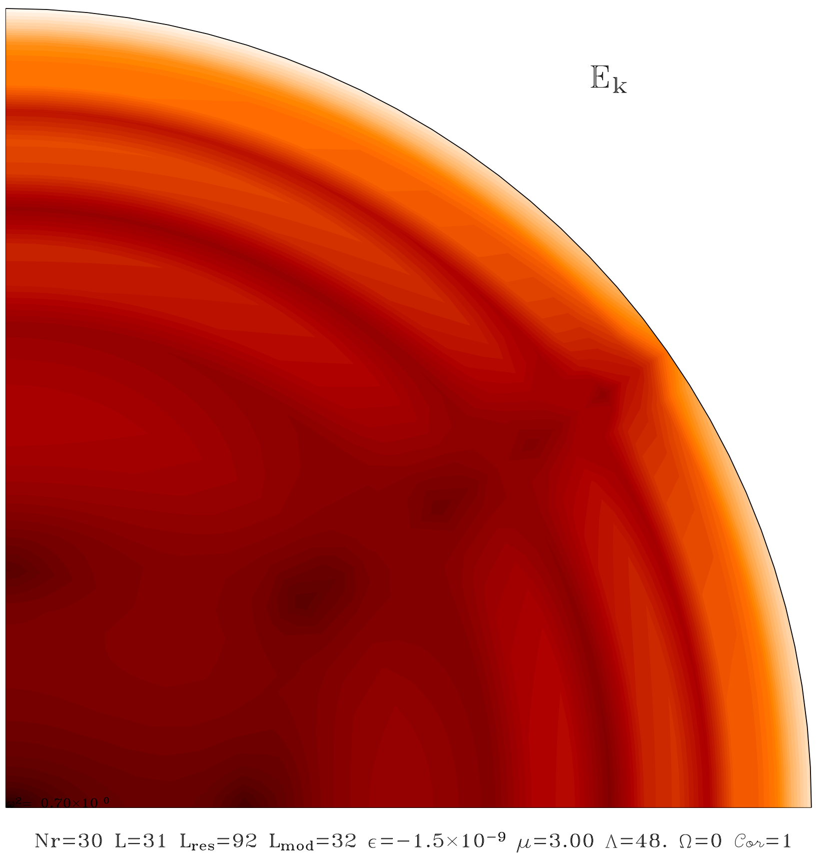
$\Rightarrow \Omega/\Omega_K \sim 0.38$



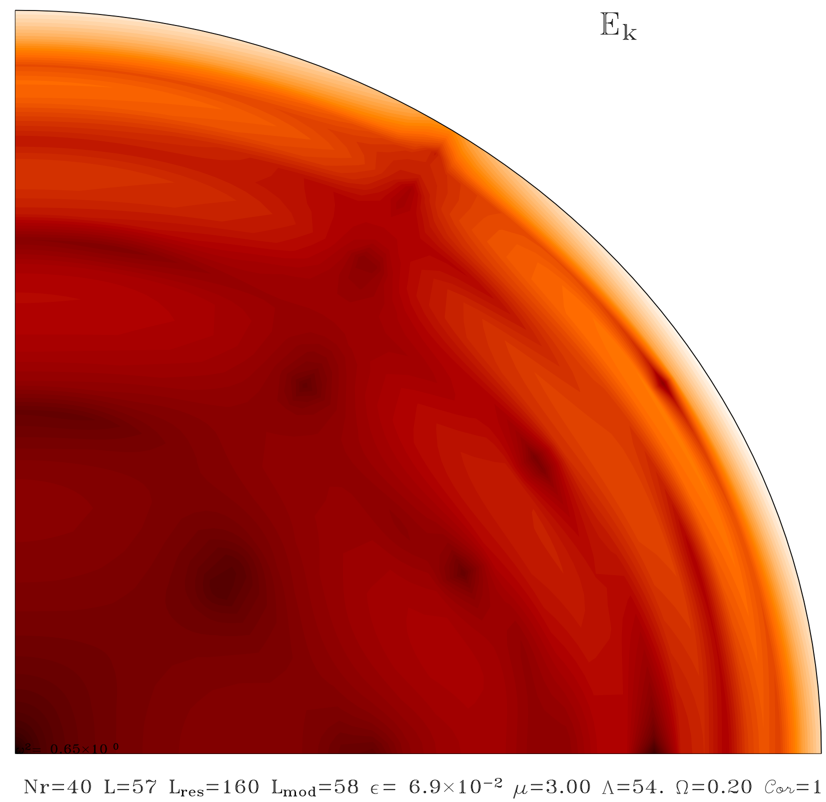
- The corresponding frequency deviation is : $-14\ \mu Hz \leq \Delta\nu \leq 23\ \mu Hz$
- The expected accuracy of Corot is :
  $\Delta\nu = 0.6\ \mu Hz$ for 20 day runs
  $\Delta\nu = 0.08\ \mu Hz$ for 150 day runs

- For a frequency deviation $|\Delta\nu| \sim 1\ \mu Hz$
  $\Omega/\Omega_K \sim 0.12$ (revised after talk)
  $v_{eq} \sim 56\ km.s^{-1}$ (revised after talk)

Oscillation mode structure

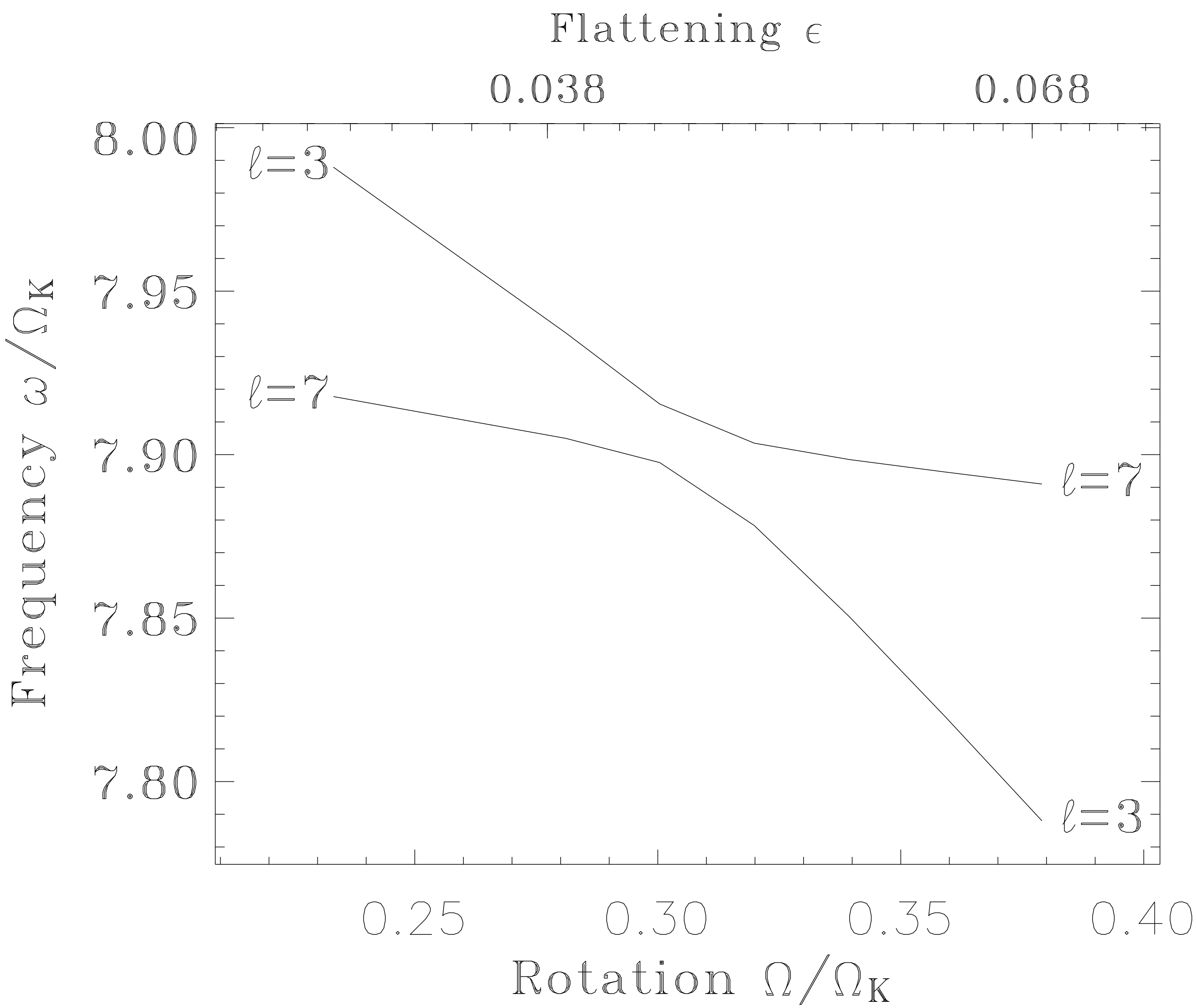


$\Omega/\Omega_K = 0.00$

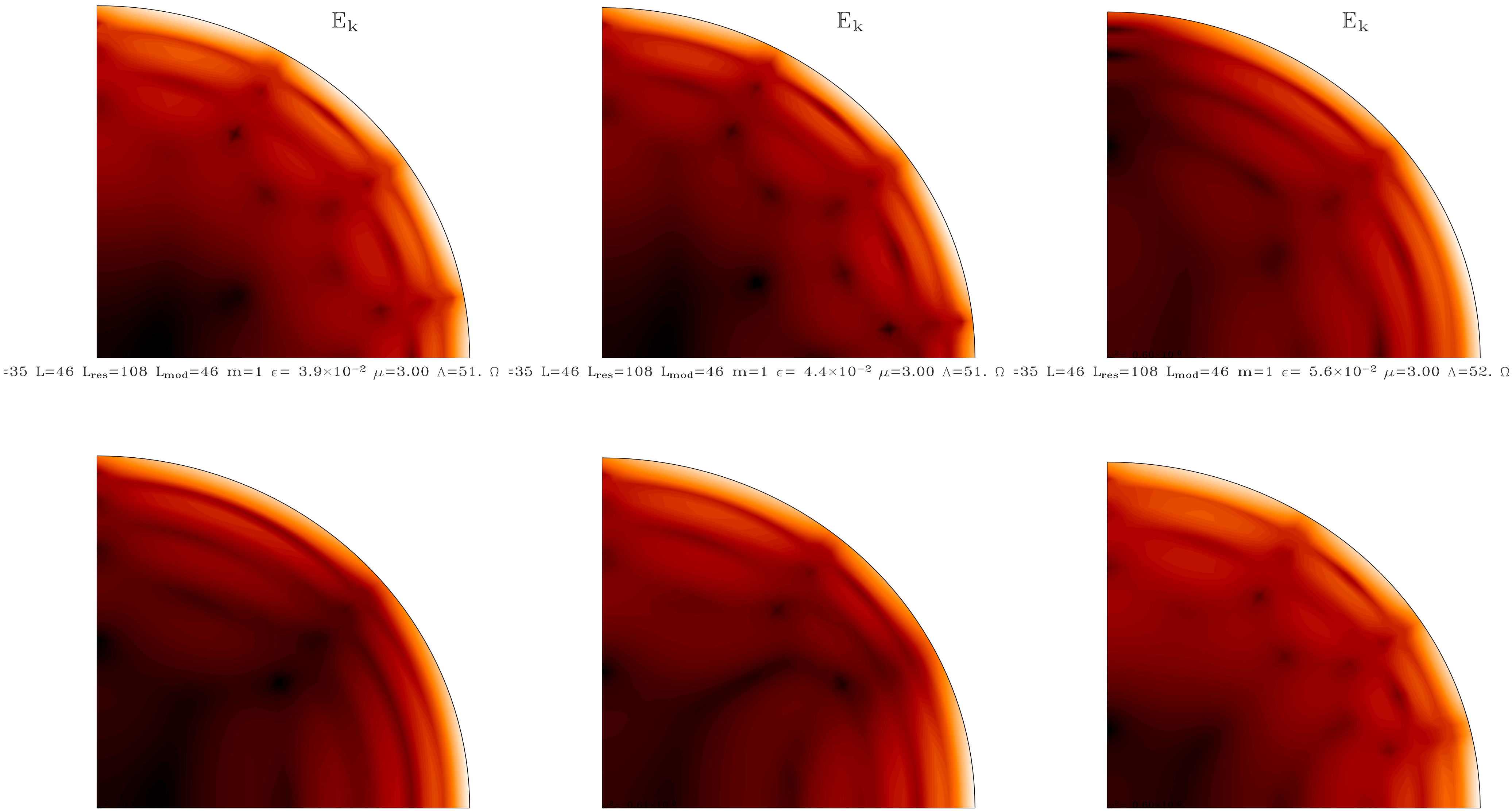


$\Omega/\Omega_K = 0.38$

An avoided crossing



An avoided crossing (2)



$\Omega/\Omega_K = 0.28$ $\Omega/\Omega_K = 0.30$ $\Omega/\Omega_K = 0.34$

Conclusion

- ✘ need for complete numerical methods for rapidly rotating stars
- ✘ future work includes :
  - comparison with 3^{rd} order perturbative methods
  - investigating higher rotation rates
  - characterising frequency spectra, and mode structures
  - trying out different equilibrium models