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Modelling of pulsations of giant stars

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ABSTRACT

Aims. We tackle the problem of interaction between convection and pulsations in giant stars.
Methods. The term $(1/\rho)\nabla \cdot \mathbf{F}'_{\rm conv}$ is used explicitly in the equations of thermal equilibrium and energy transfer. This means that the convective flux is no more considered "frozen" during the pulsation.
Results. We present the results of numerical computations of oscillation properties of a model of the G9.5 giant ϵ Oph, based on the new treatment of convection. The effects on modes stability and modes inertia are pointed out.

Key words. convection – stars: individual: ϵ Oph – stars: oscillations.

1. Introduction

Many red giants were observed by the CoRot satellite and tens oscillation frequencies was detected for each of them. In this paper we develop a tool in order to interpret the observed oscillation properties of these stars. As is well-known, a red giant star has an extended convective envelope (more than 90 percents of stellar radius), making the treatment of convection to be very important for this type of stars. In this context we included in our pulsational code the "active convection". The term "active convection" means that the convective flux is not "frozen" during the pulsation. This is an original idea with respect to the preceding models (Saio 1980, Lee 1993, Guenther 1994). Thus, a method which takes into account the variation of convective flux in time of pulsation is developed. The expression of the convective flux is written according to the "mixing length" theory (Kipenhahn 1991). The resulting equations allow us to study the interaction between convection and pulsations, a difficult question of the stellar pulsation theory. Moreover, one can also improve on the modeling of convection near the surface, but this problem needs to include 3D hydrodynamical modelling of the surface and the outer part of the convection zones of the giant stars. A number of tests are done in order to understand better the influence of the "active convection" on the theoretical frequency spectrum of giant stars compared to "frozen" convection.

2. Mathematical model

In order to obtain the eigenfunctions and the eigenfrequencies for linear, non-radial and non-adiabatic oscillations, in the case of ”active” convection, we will focus on the equations of thermal equilibrium and transfer of energy. The equation of thermal equilibrium is

$$T\frac{d\delta s}{dt} = \delta\left(\epsilon - \frac{1}{\rho}\nabla\cdot\mathbf{F}\right) \tag{1}$$

where ϵ and $\mathbf{F}$ denote, respectively, the rate per unit mass of thermonuclear energy generation and the net heat flux including, in principle, all mechanisms that may be contributing to the heat transfer, and δs denotes the Lagrangian variation of the specific entropy s. The right-hand side of above equation reads:

$$\delta\left(\epsilon - \frac{1}{\rho}\nabla\cdot\mathbf{F}\right) = \delta\epsilon + \frac{\rho'}{\rho}\mathcal{E} - \frac{1}{\rho}\nabla\cdot\mathbf{F}' - \frac{\delta r}{r}\frac{d\mathcal{E}}{d\ln r} \tag{2}$$

where $\mathcal{E} = (1/\rho)\nabla\cdot\mathbf{F}$. The energy flux $\mathbf{F}$ contains the radiative flux $\mathbf{F}_{\mathrm{rad}}$ and convective flux $\mathbf{F}_{\mathrm{conv}}$:

$$\mathbf{F} = \mathbf{F}_{\mathrm{rad}} + \mathbf{F}_{\mathrm{conv}} \tag{3}$$

where the radiative flux may be expressed using the Eddington approximation:

$$\mathbf{F}_{\mathrm{rad}} = -\frac{4\pi}{3\kappa\rho}\nabla J \tag{4}$$

with

$$J = \frac{ac}{4\pi}T^4 + \frac{1}{4\pi\kappa}T\frac{ds}{dt} \tag{5}$$

which is the expression for the mean intensity as indicated by Unno (1966); $a = 7.57 \times 10^{-15} erg cm^{-3} K^{-4}$ denotes the radiation density constant, c is the velocity of light and κ is opacity of stellar matter. We can write:

$$\frac{1}{\rho}\nabla\cdot\mathbf{F}' = \frac{1}{\rho}\nabla\cdot\mathbf{F}'_{\mathrm{rad}} + \frac{1}{\rho}\nabla\cdot\mathbf{F}'_{\mathrm{conv}} \tag{6}$$

Many of non-radial, non-adiabatic stellar oscillation codes neglect the term $(1/\rho)\nabla\cdot\mathbf{F}'_{\mathrm{conv}}$ considering that the convective flux is ”frozen” during the pulsation. In the following we wrote explicitly this term and included the expression in the energy equation.

From the expression of the radiative flux we obtain:

$$\frac{1}{\rho}\nabla\cdot\mathbf{F}'_{\mathrm{rad}} = \frac{1}{4\pi r^2\rho}\frac{\partial}{\partial r}\left[4\pi r^2 (F_r)'_{\mathrm{rad}}\right] + \frac{4\pi}{3\kappa\rho}\frac{l(l+1)}{r^2}J' \tag{7}$$

where F_r denotes the radial component of the radiative flux.

As regards the convective flux, we can write:

$$\frac{1}{\rho}\nabla\cdot\mathbf{F}'_{\mathrm{conv}} = \frac{1}{4\pi r^2\rho}\frac{\partial}{\partial r}\left[4\pi r^2 (F_r)'_{\mathrm{conv}}\right] + \frac{1}{\rho}\left[\frac{1}{r\sin\theta}\frac{\partial\left(\sin\theta\,(F_\theta)'_{\mathrm{conv}}\right)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial\left(F_\varphi\right)'_{\mathrm{conv}}}{\partial\varphi}\right] \tag{8}$$

where $(F_r)'_{\mathrm{conv}}$, $(F_\theta)'_{\mathrm{conv}}$ and $\left(F_\varphi\right)'_{\mathrm{conv}}$ denote, respectively, the radial, polar and azimuthal component of the convective flux. In what follows, because of a lack of an adequate theory for time dependent convection, we will retain only the first term in the right-hand side of the above equation. Thus, we can write:

$$\frac{1}{\rho}\nabla\cdot\mathbf{F}' = \frac{1}{4\pi r^2\rho}\frac{\partial}{\partial r}\left[4\pi r^2 (F_r)'\right] + \frac{ac}{3\kappa\rho}\frac{l(l+1)}{r^2}T^4\left[4\frac{\delta T}{T} + \frac{1}{4\pi\kappa}\frac{d\delta s}{dt} - 4\frac{d\ln T}{d\ln r}\left(\frac{\delta r}{r}\right)\right] \tag{9}$$

where F_r denotes the radial component of total flux: $F_r = (F_r)_{\rm rad} + (F_r)_{\rm conv}$. From the expression of the radiative flux we obtain:

$$(F_r)_{\rm rad} = -\frac{4acT^4}{3\kappa\rho r}\frac{d\ln T}{d\ln r} \tag{10}$$

As regards the convective flux, we will use the expression given by the "mixing length" theory (Kippenhahn 1991):

$$(F_r)_{\rm conv} = \rho c_{\rm P} T\,\sqrt{g\delta}\,\frac{\alpha^2_{\rm MLT}}{4\sqrt{2}} H_{\rm P}^{1/2}\,(\nabla - \nabla_e)^{3/2} \tag{11}$$

where $\alpha_{\rm MLT}$ is defined by $l_m = \alpha_{\rm MLT} H_{\rm P}$, l_m being the mixing length, $H_{\rm P} = -1/(d\ln P/dr) = P/\rho g$ denotes the pressure scale, $\delta = -(d\ln\rho/d\ln T)$. The quantity ∇_e describes the variation of T in the convective element in time of its motion, its position measured with help of P.

In that following we calculated the relative Lagrangian variation of the radial component of the radiative and convective flux, and we wrote explicitly the equations of thermal equilibrium and transfer of energy. Finally, the system of differential equations describing non-adiabatic oscillations is solved with proper boundary conditions. The whole system of differential equations and the corresponding boundary conditions will be written down in a forthcoming paper.

3. Case of ϵ Oph

In order to have a better insight on the effect of the "active" convection on pulsational stability in red giant stars compared to "frozen" convection we calculated radial and strongly trapped non-radial modes with mode degree l up to 3 of an stellar model for ϵ Oph obtained by Pricopi (2008). The model matches (in $\pm 1\sigma$) 16 photometric frequencies observed by MOST satellite (see Table 1). The fundamental stellar parameters of this model are: $M = 2.03 M_\odot$, $T_{\rm eff} = 4892$K, $L = 59.13 L_\odot$, $R = 10.76 R_\odot$ and $age = 0.73 Gyr$. The model show us that ϵ Oph has an convective envelope extended on 91 percents of stellar radius. Thus, we expect that the effect of variation of convective flux during the pulsation on the mode stability in giant stars to be important.

4. Conclusions

1. The effect of the term $(1/\rho)\nabla\cdot\mathbf{F}'_{\rm conv}$ on modes inertia becomes important for frequencies higher than 50μHz in the case of a model of ϵ Oph. The radial and non-radial strongly trapped modes with mode degree $l = 1, 2$ have a lower inertia (they are more detectable) than in the "classic" case in which the convection is considered "frozen" during the pulsations.
2. In the case of "frozen" convection we have found that all the radial modes are stable ($\eta < 0$) while all the non-radial strongly trapped modes (in the frequency range that was detected, i.e. $< 100\mu$Hz) are unstable ($\eta > 0$).
3. If the effect of the term $(1/\rho)\nabla\cdot\mathbf{F}'_{\rm conv}$ is taken into account in the stability analysis we have found that some of radial modes with low frequency ($< 40\mu$Hz) are stable while the rest of them becomes unstable. As regards the non-radial strongly trapped modes (in the frequency range that was detected, i.e. $< 100\mu$Hz) some of them becomes stable. The linear growth rates of these non-radial modes are similar to those of corresponding radial modes.
4. If we focus on the 16 photometric frequencies observed by MOST satellite that are matched in $\pm 1\sigma$ by the model (Table 1) we see that 3 of 4 radial modes becomes unstable while 2 non-radial

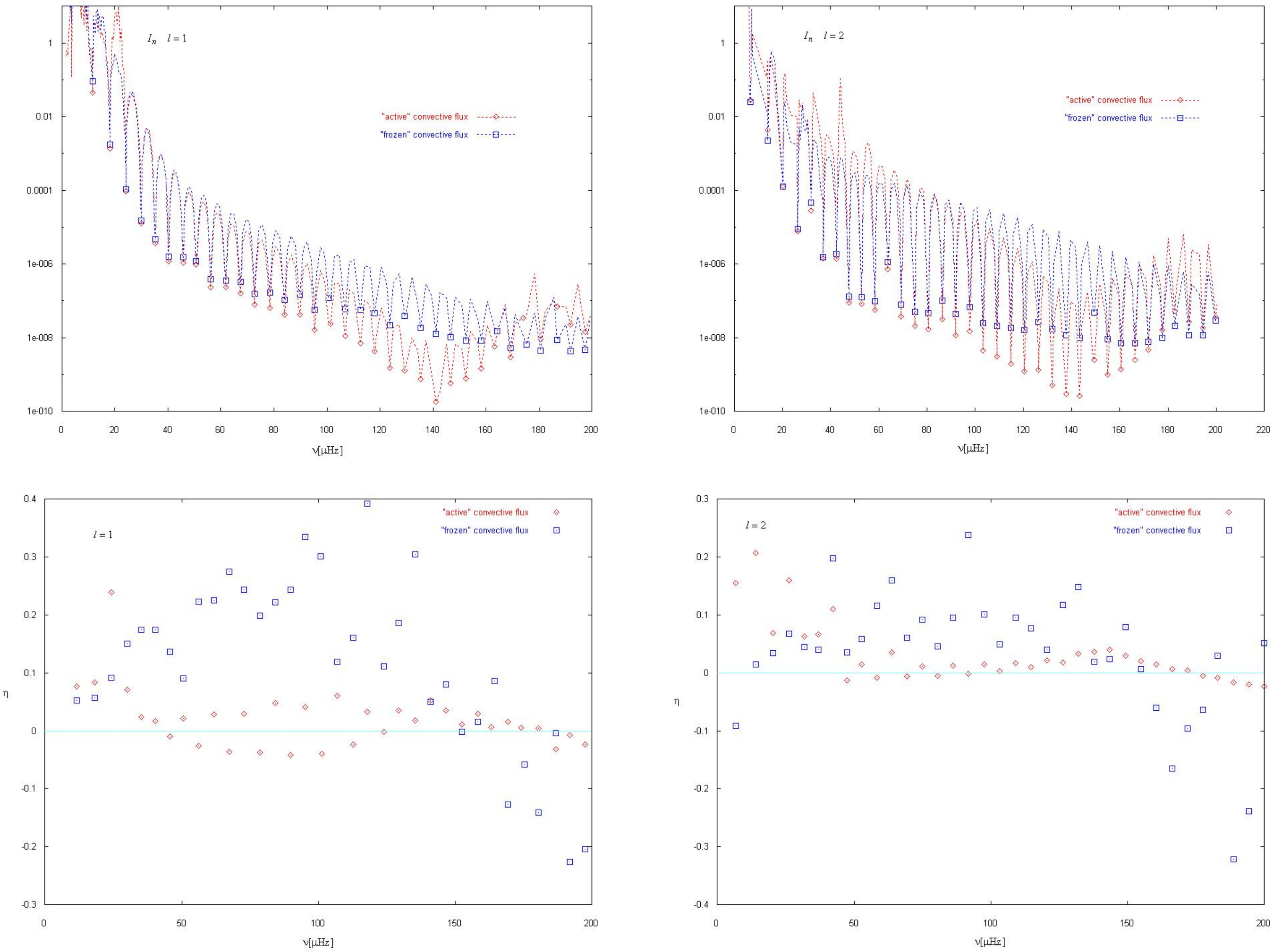

Fig. 1. Modal inertia in units of $3MR^3$, plotted against frequency (the two upper panels). Red color is used for "active" convective flux and blue color for "frozen" convective flux. The symbols are displayed only for those modes that are locally most trapped. We clearly see that the effect of "active" convective flux is important at frequencies $\geq 50\mu$Hz and inertia has a minima at 140μHz in the case of "active" convection for all values of mode degree l. The modes with frequencies below 50μHz seem to be less affected. Coefficient of stability ($\eta = W/\int_0^1 (dW/dx)dx$), is plotted against frequency the two bottom panels. The symbols are displayed only for those modes that are locally most trapped. We clearly see a local positive maxima at 140μHz in the case of "active" convection for all values of mode degree l.

modes (one with $l = 1$ and one with $l = 2$) become stable in the case of "active" convection. So, we are still unable to explain the observed oscillation properties of ϵ Oph in terms of mode instability. It is expected that many of them are stochastic excited by turbulent convection.

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Table 1. The 16 photometric frequencies observed by MOST satellite (Kallinger 2008) that are matched in $\pm 1\sigma$ by the model. For each frequency, the coefficient of stability and the inertia in terms of $3MR^3$ are written down both for "frozen" convective flux (index 1) and "active" convective flux (index 2).

	$\nu_{\rm obs}[\mu{\rm Hz}]$	$\nu_{\rm calc}[\mu{\rm Hz}]$	η^1	η^2	I_n^1	I_n^2
f_1	8.49 ± 0.06	$8.47(l = 3)$	+	+	7.08×10^{-4}	8.08×10^{-4}
f_2	13.99 ± 0.09	$13.96(l = 2)$	+	+	2.28×10^{-3}	4.42×10^{-3}
f_3	15.20 ± 0.16	$15.11(l = 0)$	-	-	4.24×10^{-5}	4.01×10^{-5}
f_4	22.14 ± 0.18	$22.01(l = 3)$	+	+	5.66×10^{-6}	5.51×10^{-6}
f_5	24.24 ± 0.12	$24.20(l = 1)$	+	+	1.08×10^{-4}	9.56×10^{-5}
f_6	32.57 ± 0.11	$32.67(l = 0)$	-	+	2.23×10^{-7}	1.86×10^{-7}
f_7	36.97 ± 0.15	$36.95(l = 2)$	+	+	1.51×10^{-6}	1.38×10^{-6}
f_8	38.52 ± 0.09	$38.60(l = 3)$	+	+	1.23×10^{-6}	8.92×10^{-6}
f_9	40.16 ± 0.16	$40.32(l = 1)$	+	+	1.57×10^{-6}	1.20×10^{-6}
f_{10}	48.11 ± 0.09	$48.15(l = 0)$	-	+	3.20×10^{-8}	2.22×10^{-8}
f_{11}	54.69 ± 0.14	$54.63(l = 3)$	+	+	3.73×10^{-8}	2.34×10^{-8}
f_{12}	61.92 ± 0.14	$61.85(l = 1)$	+	+	3.57×10^{-7}	2.30×10^{-7}
f_{13}	64.39 ± 0.12	$64.34(l = 0)$	-	+	1.94×10^{-8}	1.03×10^{-8}
f_{14}	67.49 ± 0.17	$67.34(l = 1)$	+	-	3.20×10^{-7}	1.58×10^{-7}
f_{15}	69.27 ± 0.13	$69.31(l = 2)$	+	-	8.02×10^{-8}	3.75×10^{-8}
f_{16}	93.74 ± 0.18	$93.91(l = 3)$	+	+	1.58×10^{-8}	3.98×10^{-9}

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