On extracting signatures of small convective cores from space-based data

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Can we predict the signature of a small convective core on the eigenfrequencies?

Yes. Cunha and Metcalfe (2007, ApJ 666, 413) derived the theoretical perturbation to the oscillation frequencies of radial modes due to the presence of a small convective core. To do that, they carried out a variational study using an asymptotic expression for the displacement that is valid close to the inner turning point of the oscillations. In their calculation they assumed that the effect of the convective core on the frequencies arises principally from the sharp variation of the sound speed at the edge of the core.

The theoretical expression derived by the authors is as follows:

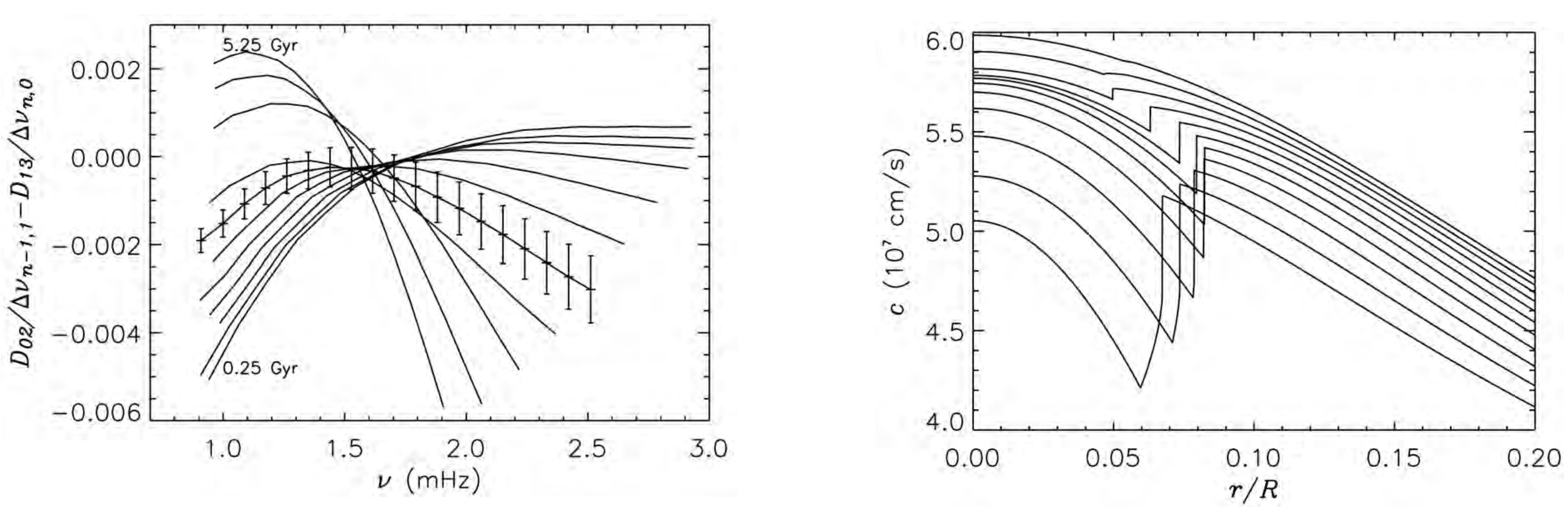
$$\delta\nu^c \approx -\frac{A_\delta}{2\nu I}\left[\frac{|K| r^3 \rho c^2 \xi^2}{|x|^{1/2}}\right]_{x=0}\left[\frac{r\left(\frac{4g}{4\pi^2}+\nu^2 r\right)}{c^2|K|^2}\right]_{x=x_{\rm d}} \times \left[|x|x - \frac{A_{\rm i}'(0)}{A_{\rm i}(0)}|x|x^2 + \frac{1}{3}\frac{A_{\rm i}'(0)^2}{A_{\rm i}(0)^2}|x|x^3\right]_{x=x_{\rm d}}, \qquad (1)$$

The meaning of the symbols is defined in the paper mentioned above (attached to the poster). The important aspects to know about the variables in this equation are the following:

1) The independent variable x is a function of frequency. It is equal to zero at the inner turning point, negative below it (smaller radius), and positive above it. x_d is the value taken by x at the edge of the convective core and is also a function of frequency.

2) The amplitude A_δ is a positive constant related to the sound speed sharp increase at the edge of the convective core.

It is clear from the above that the frequency perturbation is a function of frequency. It is zero for a mode whose turning point coincides with the edge of the core, positive for modes of lower frequency, and negative for modes of higher frequency. This perturbation is displayed in Figure 2 (left panel) for a set of equilibrium models.



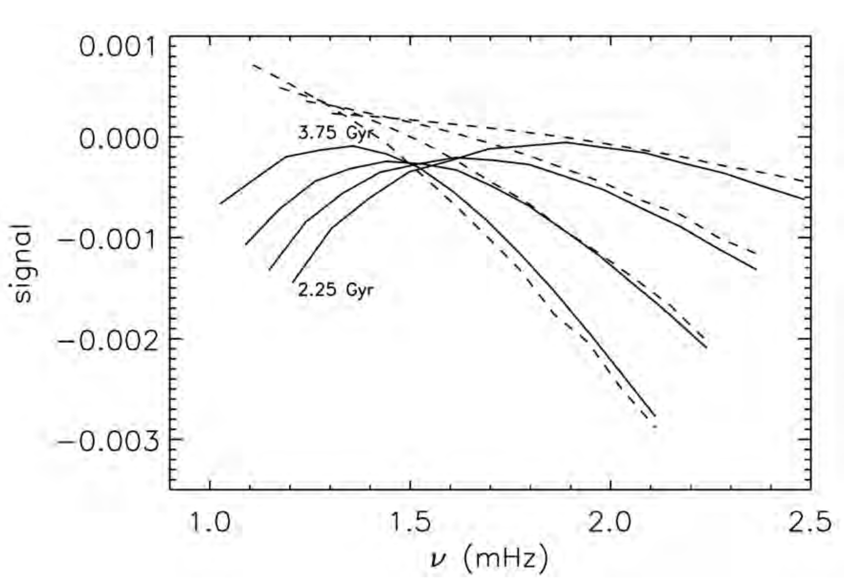


Figure 1

***Upper left panel**:* Frequency differences - lhs of eq. (2) - for 1.3 M$_\odot$ models with ages ranging from 0.25 Gyr to 5.25 Gyr (intervals of 0.5 Gyr). Error bars assume a relative error in the individual frequencies of 10^{-4}.

Lower left panel: comparison between the theoretical signal - eq. (1) normalized according to the rhs of eq. (2) - (dashed lines) and the frequency differences mentioned above (full lines) for 1.3 M$_\odot$ models with ages ranging from 2.25 Gyr to 3.75 Gyr.

Upper right panel: The sound speed profile near the core for the 1.3 M$_\odot$ set of models, with ages between 0.25 Gyr (uppermost) to 5.25 Gy (lowermost).

Can we isolate the core's signature through a particular combination of the eigenfrequencies?

Yes. In the same work, the authors have shown that, asymptotically:

$$\frac{D_{02}}{\Delta\nu_{n-1,1}} - \frac{D_{13}}{\Delta\nu_{n,0}} \approx \frac{\delta\nu^c}{6\,\Delta\nu_{n-1,1}} \qquad (2)$$

where $D_{l,l+2}$ are the scaled small separations and $\Delta\nu_{n,l}$ are the large separations, n and l being the mode radial order and degree, respectively.

By combining the oscillation frequencies according to the lhs of eq. (2), we can recover, in the asymptotic regime of high frequencies, the frequency perturbations due to the sharp sound speed variation at the edge of the core. Moreover, the slope of the proposed quantity at high frequencies provides a direct measure of the 'jump' of the sound speed at the edge of the core.

Figure 1 illustrates these results, for a set of models of 1.3 M$_\odot$, at different ages during the Main Sequence. Models were computed with ASTEC (Christensen-Dalsgaard, 2008, ApSS 316,13).

Can we expect to detect the core's signature in the frequencies derived from space-based data?

Yes. The diagnostic tool suggested by eq. (2) relies on the identification of modes of degrees l=0,1,2 and 3. Clearly, that is a problem for photometric data, given the difficulty in detecting the l=3 modes. Thus, we turn our attention to the small frequency separations involving modes of degrees l=0,2. These can be written as:

$$\delta\nu_{02} = \delta\nu_{02}^s + \delta\nu^c \qquad (3)$$

where the superscripts '*s*' and '*c*' correspond, respectively, to the smooth part of the small separations, and the perturbation to it due to the sharp variation of the sound speed at the edge of the core. In eq. (3) we have assumed, as Cunha and Metcalfe, that the l=0 modes are affected by the edge of the core much more strongly than all other modes. Also, we have assumed that sharp structural variations in the stellar envelope affect all low degree modes in a similar way.

We want to know:

1) If $\boldsymbol{\delta\nu^c}$ is sufficiently large to have a detectable effect on $\boldsymbol{\delta\nu_{02}}$;

2) If it is possible to isolate $\boldsymbol{\delta\nu^c}$, once we have observed $\boldsymbol{\delta\nu_{02}}$ as a function of frequency. For that, the functional form of $\boldsymbol{\delta\nu^c}$ needs to be distinct from the functional form of $\boldsymbol{\delta\nu_{02}^s}$.

Figure 2 illustrates what may be expected for the case of 1.3 M$_\odot$ models at different ages in the Main Sequence. Note that the shift in the small separations is larger than 1μHz for some of the models and would be even larger for more evolved models (cf. Figure 3).

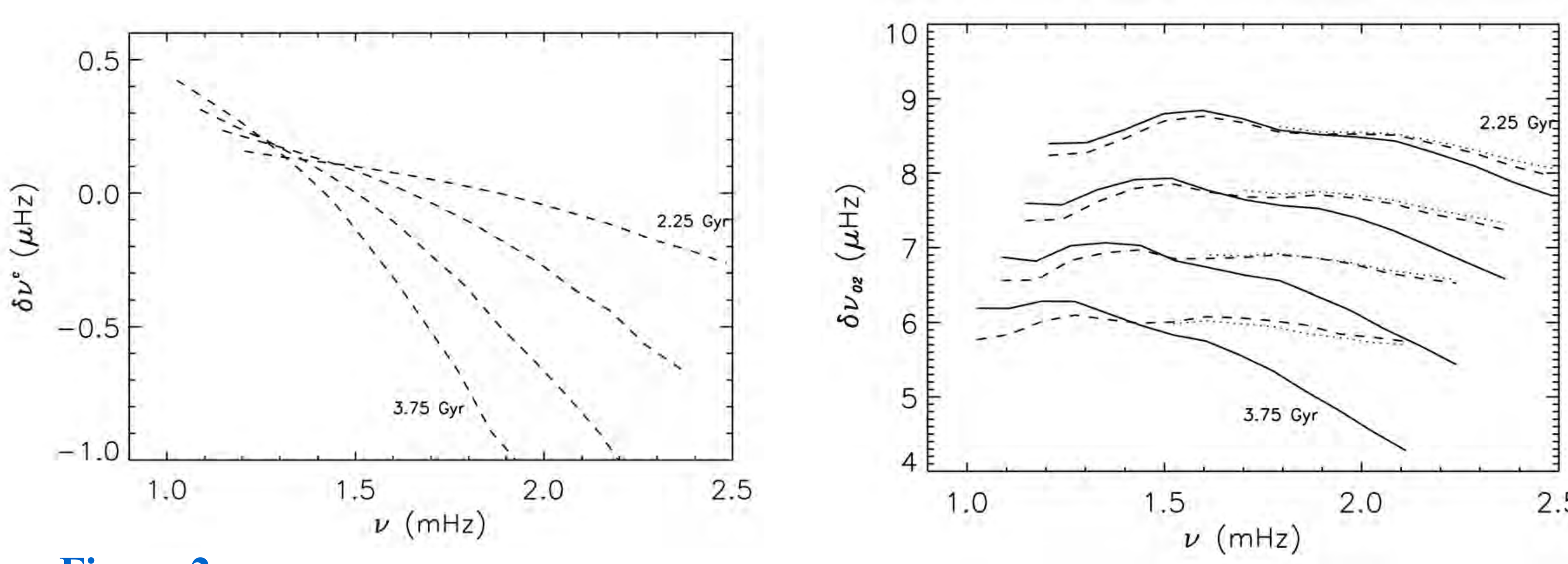


Figure 2

***Left panel**:* Frequency perturbations due to the border of the core (cf. eq. (1)), for 1.3 M$_\odot$ models. Age range: 2.25 Gyr to 3.75 Gyr (in intervals of 0.5 Gyr).

Right panel: Small separations (l=0,2;full lines) and the same quantity after subtracting $\boldsymbol{\delta\nu^c}$ (dashed lines) for the 1.3 M$_\odot$ models. The dotted lines show the smooth component $\boldsymbol{\delta\nu_{02}^s}$ estimated at high frequencies through the scaled small separations $6^*D_{13}{}^*\Delta\nu_{n-1,1}/\Delta\nu_{n,0}$. Note that D_{13} involve only modes of l=1,3 and, thus, are not significantly affected by the edge of the core.

What will we learn about the core if this signature is detected?

Since the signature is a consequence of the sharp variation of the sound speed at the edge of the core, the frequency derivative of the small separations provides a measure of the amplitude of that sharp variation (and, indirectly, of the star's age). This is illustrated in Figure 3.

Moreover, Cunha and Metcalfe have shown that other properties of the frequency differences given by eq (2) are sensitive to the size of the convective core, the star's age, and the stage at which the core begins to contract. We shall explore whether that is the case also of the signature left on the small separations.

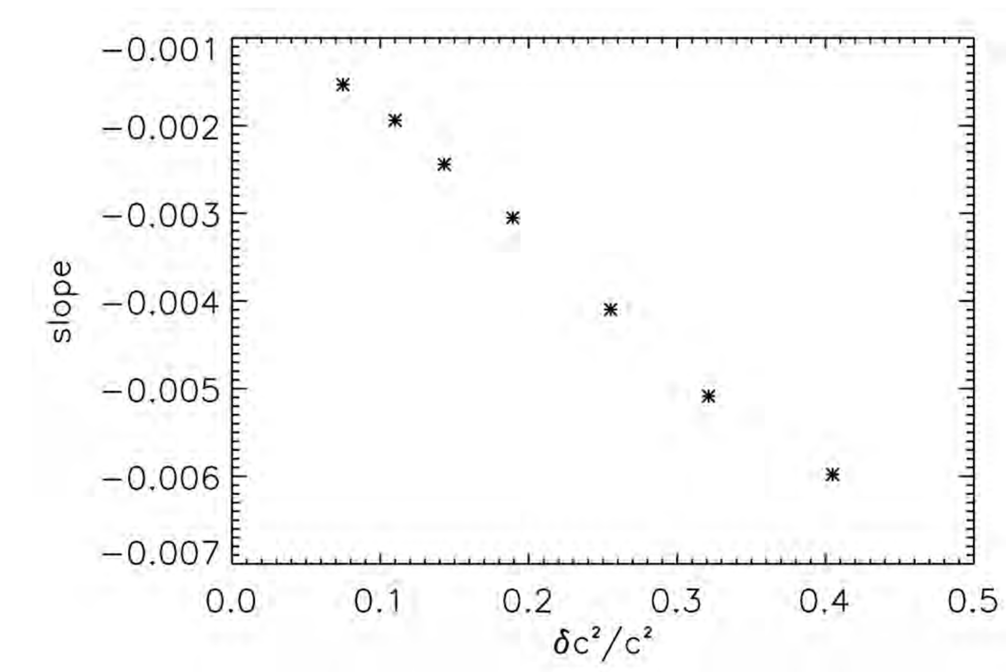


Figure 3

The slope of the small separations calculated a fixed (hight) n, as function of the relative 'jump' in the square of the sound speed at the edge of the core for 1.3 M$_\odot$ models. Age range: 2.25 Gyr (upper-left) to 5.25 Gy (lower-right) (in intervals of 0.5 Gyr).