

Narrow frequency windowed autocorrelations as a diagnostic of solar-like stars

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ABSTRACT

Aims. To investigate the diagnostic potential of narrow frequency windowed autocorrelation as a tool for probing the properties of solar-like oscillating stars when the signal to noise is small and the determination of frequencies is not possible or is subject to large uncertainties.

Methods. Theoretical analysis, phase-shifts, modelling, and data analysis.

Results. The power in the narrow windowed autocorrelation can reveal the variation with frequency of the large separations $\Delta(\nu)$ and the Half Large separations Δ_{01}, Δ_{10} . This technique is applied to the CoRoT p-mode oscillators, HD49933, HD175726, HD181420, and HD181906. Theoretical analysis and modelling are presented to illustrate the technique.

Key words. stars: oscillations - methods: analytical - methods: data analysis

1. Introduction

For solar-like stars a reliable determination of p-mode frequencies from power spectra is not always possible since the amplitudes of the stochastically excited modes are very small, giving low signal/noise. For F stars observed by CoRoT¹ the line widths are large which hinders the reliable determination of frequencies and for some stars, particularly HD175726, individual frequencies are exceedingly difficult to extract. I here consider an alternative approach which may be useful when faced with poor quality data, namely the use of the autocorrelation of the time series. As shown by Roxburgh and Vorontsov (2006) by adding noise to a solar power spectrum, this has diagnostic potential when faced with noisy data.

The location of major peaks in the autocorrelation function of an observational time series provides an estimate of the average value of the large frequency spacings (Δ) between modes of consecutive radial order (cf Gabriel et al, 1998, R&V 2006); this gives a measure of the stellar acoustic diameter. But the large separations are not independent of frequency ν , or degree ℓ , the variation is primarily due to the quasi-periodic signal in the frequencies caused by the HeII ionisation layers, Other factors influencing the large separations are the different contributions of the inner layers to frequencies of different degree ℓ and the additional quasi-periodic signal due to boundaries of convective envelopes and cores, and any other regions of sharp change in the acoustic structure of the star.

I here explore to what extent one can determine the frequency dependence of the large separations by using narrow frequency windowed autocorrelations of the time series.

2. The autocorrelation power and the large separations and HD49933

Consider a wave packet produced near the surface of the star - it propagates to the far side of the star in time $2T$, where T is the acoustic radius of the star, where it is reflected back arriving at (near) the point of emission at $4T$. Thus one expects a peak in the autocorrelation at $4T = 2/\Delta$ where Δ is some mean large separation. Since the actual ray path and the location of the reflecting layer depend on the star's structure, and on frequency and degree, the round trip travel time is not independent of ν, ℓ , and the large separations $D_0 = \nu_{n,0} - \nu_{n-1,0}$ and $D_1 = \nu_{n,1} - \nu_{n-1,1}$ vary with both frequency and degree.

In what follows I work in terms of the power spectrum of the frequency power spectrum rather than with the autocorrelation itself. This is in fact the square the envelope of the actual autocorrelation function but has the advantage that it removes the rapid oscillation on the time scale of the oscillations. I take the power spectrum of a given time series, window in the frequency domain and take the power spectrum of this windowed segment.

As an example of the procedure I use the data on the F5V star HD49933 observed for 60 days in the initial run of CoRoT (see Appourchaux et al 2008). The time series was first filtered in the time domain by subtracting off a running 6 hour average and gaps (mostly passes through the South Atlantic anomaly) and other "dodgy data" were replaced by Gaussian noise with standard deviation of the filtered time series. The power spectrum of this filtered time series (with peaks at harmonics of the satellite orbital period removed) was then filtered with a cosine window between 1200 – 2500 μ Hz. and the power spectrum of this power spectrum gave the autocorrelation power shown in Fig 1a.

The first peak occurs at a time $t = 6.45$ hours corresponding to a mean large separation of 86.1 μ Hz. The rapid decrease in the height of successive peaks indicates a the short life time of the modes; this is to be expected since the line widths in the power spectrum are quite broad (Appourchaux et al 2008). One can just

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¹ The CoRoT space mission, launched on 2006 December 27, was developed and is operated by CNES, with the participation of the Science Programmes of ESA, ESA's RSSD, Austria, Belgium, Brazil, Germany and Spain.

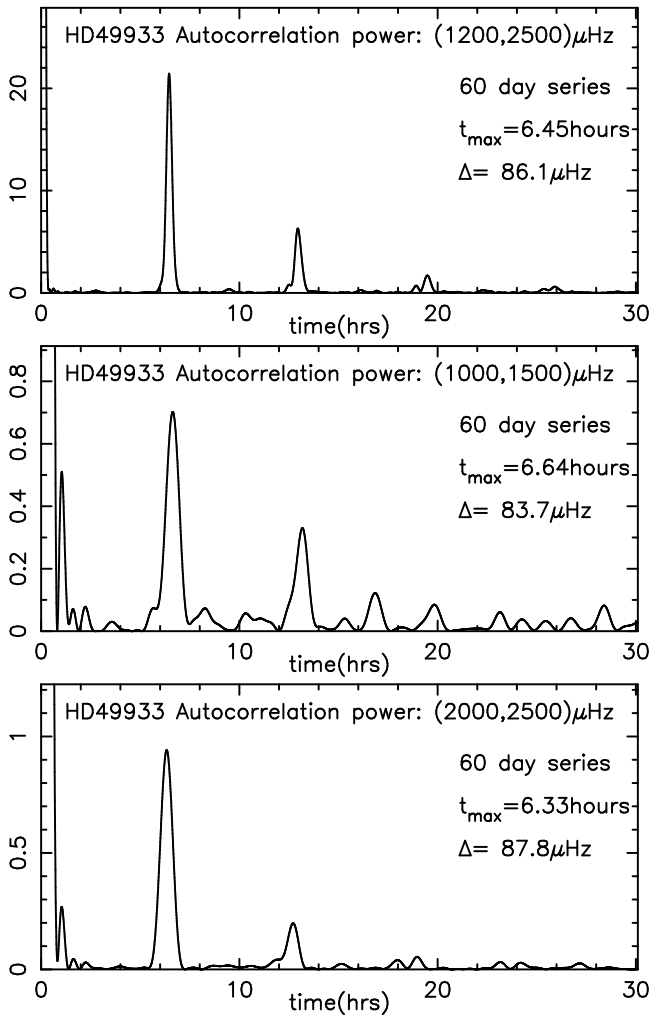


Fig. 1. Frequency windowed autocorrelation power for HD49933 (scaled): a) wide windowed between 1200 – 2500 μHz , b) narrow windows 1000 – 1500 μHz , c) narrow window 2000 – 2500 μHz .

detect the peaks at the mid points of the major peaks which are caused by the small separation (R&V 2006).

Figs 1b and 1c show the autocorrelation power for two independent narrower windows of 500 μHz . These again show the characteristic maxima near 6.45 hours but they are not at exactly the same values. This suggests one may be able to extract more detailed information on the frequency variation of Δ by using narrower windows.

I then took two sets of narrower windows of ± 200 and $\pm 400 \mu\text{Hz}$ centred on a frequency ν and moved the windows through the frequency range 1200 – 2500 μHz in steps of 5 μHz . I located the peaks t_k around the value of 6.45 hours, and hence a local value of the large separations as $\Delta(\nu) = 1/2t_k$. The results are shown in Figure 2a. Superimposed on these curves are the values of the large separations (and their formal errors) as determined by Appourchaux et al (2008). I note here that the extraction of frequencies, even for HD49933, is fraught with difficulties and different approaches to frequency extraction give different results.

The dotted curve in Fig 2a for a window of $\pm 200 \mu\text{Hz}$ shows a small scale period oscillation. This is much enhanced with a narrower window. Fig 2b shows the results of using a window of $\pm 100 \mu\text{Hz}$ moving through the frequency range. Note that the periodic modulation has a period $\approx 43 \mu\text{Hz}$, that is half of the mean

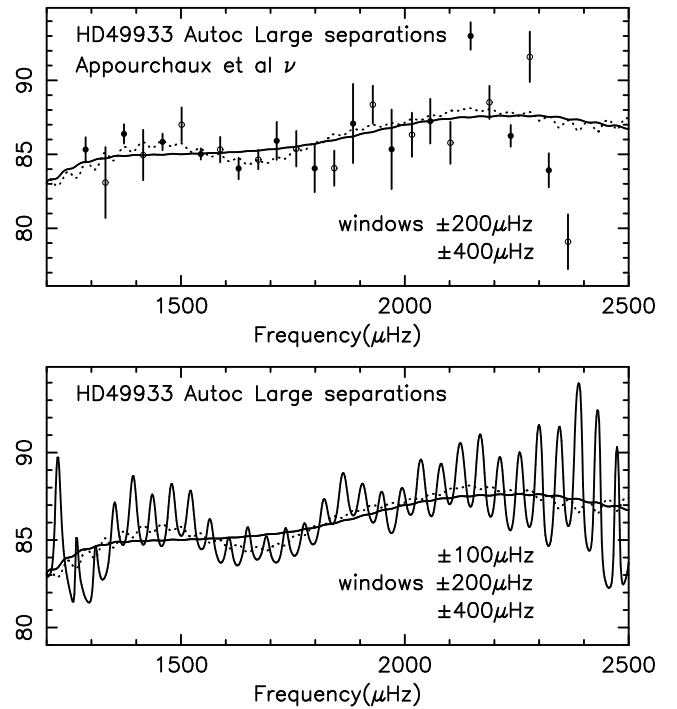


Fig. 2. a). Variation of Large separation with frequency for HD49933 for 2 different windows and superimposed the large separations with 1σ error bars as determined by Appourchaux et al (2008). b) Variation of Large separation with frequency for HD49933 for very narrow window of $\pm 100 \mu\text{Hz}$.

value of the large separations. As shown below this is because the "half large separations"

$$\Delta_{10}(n) = \nu_{n,1} - \nu_{n,0}, \quad \Delta_{01}(n) = \nu_{n,0} - \nu_{n-1,1}$$

are not expected to be equal and, with a narrow window, the resulting autocorrelation time depends on whether an $\ell = 0, 1$ or $\ell = 1, 0$ pair is in the central part of the window. The signal is also influenced by the $\ell = 0, 2$ small separations but may still provide a useful diagnostic of the internal structure of the star.

3. HD175726: A low signal to noise example

HD175726 is a F9/G0 star observed for 27 days in the first short run of CoRoT. The power spectrum is shown in Fig 3a, individual p-modes cannot be seen in the spectrum but, as shown by Michel et al (2008), this star has a slight excess of power in the range p-mode range 1000 – 3000 μHz , and some broad peaks are discernible in a boxcar of the spectrum - as shown in Fig 3b for a box width of 15 μHz . It is therefore worth while to look for evidence of the variation of large separations with frequency using the narrow windowed autocorrelation - indeed it was the challenge presented by this star that initiated the research reported here (Roxburgh 2008). Details of the properties of HD175726 are given in Mosser, Roxburgh et al (2009) along with some results similar to those reported here.

Again the long period variation of the time series was filtered out by subtracting off a 6 hour running average and the gaps and "dodgy data" filled with uncorrelated noise. Residual peaks at harmonics of the orbital period were set to zero in the power spectrum. The autocorrelation power (power spectrum of the power spectrum (filtered by a cosine window between 1000 – 3000 μHz) is shown in Fig 3c. The first significant peak

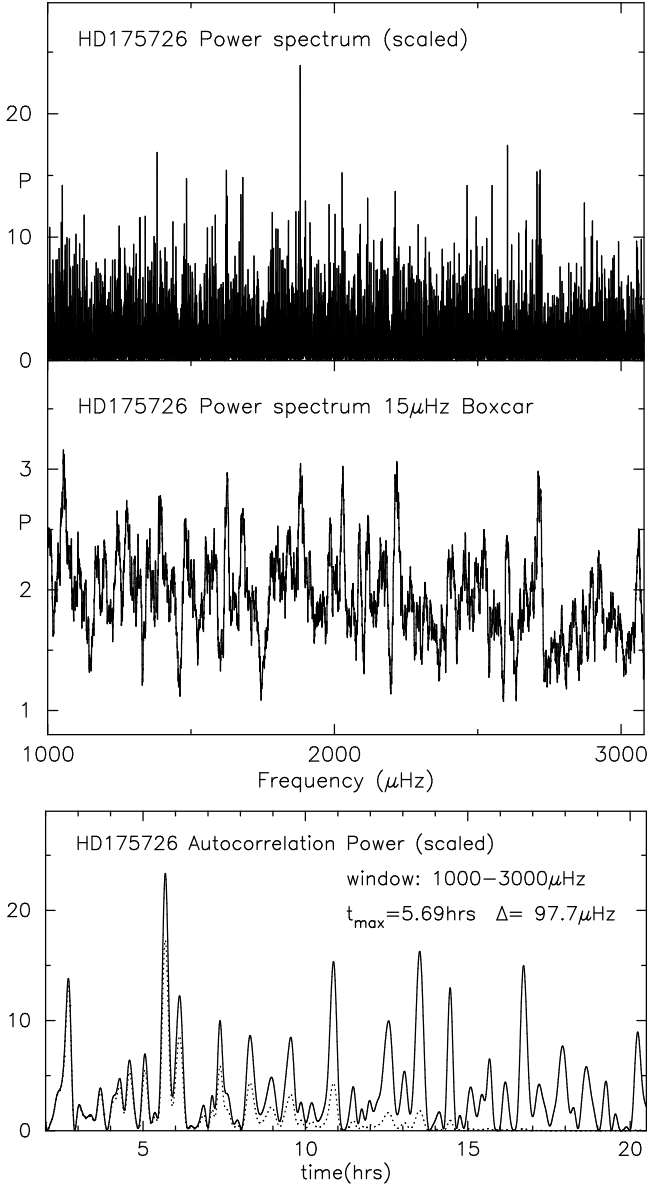


Fig. 3. a) Power spectra for HD175726: b) result of applying a boxcar of $15\mu\text{Hz}$ full width. c) Autocorrelation power for HD175726 for very wide window of $\pm 1000\mu\text{Hz}$. The dominant peak is at a correlation time of 5.69 hrs corresponding to a large separation of $97.7\mu\text{Hz}$.

away from zero is at $t = 5.69$ hrs giving a mean large separation $\Delta = 1/2t = 97.7\mu\text{Hz}$. Also shown in Fig 3c as the dotted line is the autocorrelation power obtained using the boxcar power spectrum shown in Fig 3b. It should be stressed that we are here fighting to extract a signal from the noise and the results will differ somewhat depending on how one "massages" the data: filling gaps and "dodgy data" and suppressing harmonics of the orbital period. Nevertheless the general behaviour is found with different procedures (see Mosser, Roxburgh et al 2009).

With this information I then took a set of narrow windows centred on frequencies ν in the range $1500 - 2500\mu\text{Hz}$ with half width $\delta\nu = 300, 400, 500, \mu\text{Hz}$ and identified the peak near 5.7 hours and hence a local value of $\Delta(\nu)$. With a mean large separation of $\approx 98\mu\text{Hz}$ the windows were wide enough to ensure that (if they exist) there are several $\ell = 0, 1$ modes within the window (6 - 10) which should suppress the variation between pairs of modes and give a local average of the large separa-

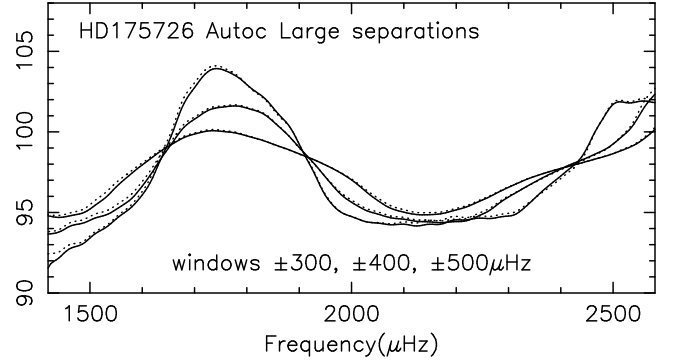


Fig. 4. Variation of Large separation with frequency for HD175726 for a set of narrow window of $\pm 300, 400, 500\mu\text{Hz}$.

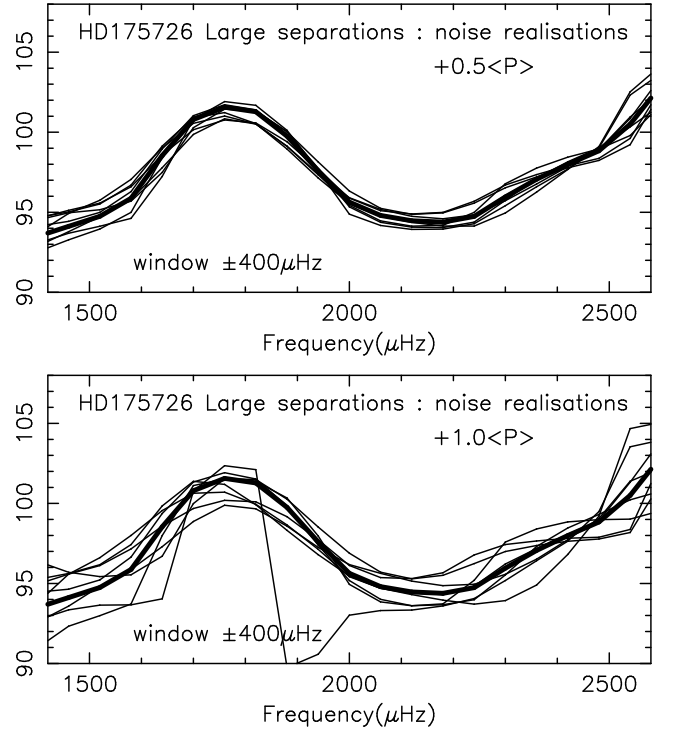


Fig. 5. Autocorrelation power for HD175726 cosine windowed with half width $400\mu\text{Hz}$, with exponential noise added to the power spectrum: a) $0.5 < P >$, b) $1.0 < P >$

tion. The results are shown in Fig. 4; all 3 curves show a similar quasi-periodic behaviour but the smaller the window the larger the amplitude of the variation. This is to be expected since the larger windows span a larger range around the peaks and troughs and therefore reduce the amplitude of the quasi-periodic variation. Such a quasi-periodic behaviour is caused by the variation in the surface phase shift $\alpha(\nu)$ due to the HeII ionisation zone, (Vorontsov & Zharkov, 1988,9, Brodskii & Vorontsov 1989, R&V 1994), and also by the internal phase shifts $\delta_i(\nu)$ (Roxburgh 2009). With a certain amount of optimism I estimate a quasi-period of order $\sim 1000\mu\text{Hz}$ which corresponds to an acoustic depth of $\sim 500\text{secs.}$. Not unreasonable for such a star.

To test the significance of the result I added exponentially distributed noise to the power spectrum with a mean of 0.5 and 1.0 times the average power $\langle P \rangle$ in the frequency range $1500 - 2500\mu\text{Hz}$, and repeated the calculations for a $\pm 400\mu\text{Hz}$ window; the results are shown in Fig 5a,b, the thick line being the result with no added noise. These results suggest that the signal is there - but this does not of course prove that it accurately reproduces the variation $\Delta(\nu)$. Estimates of the error on the autocorrelation

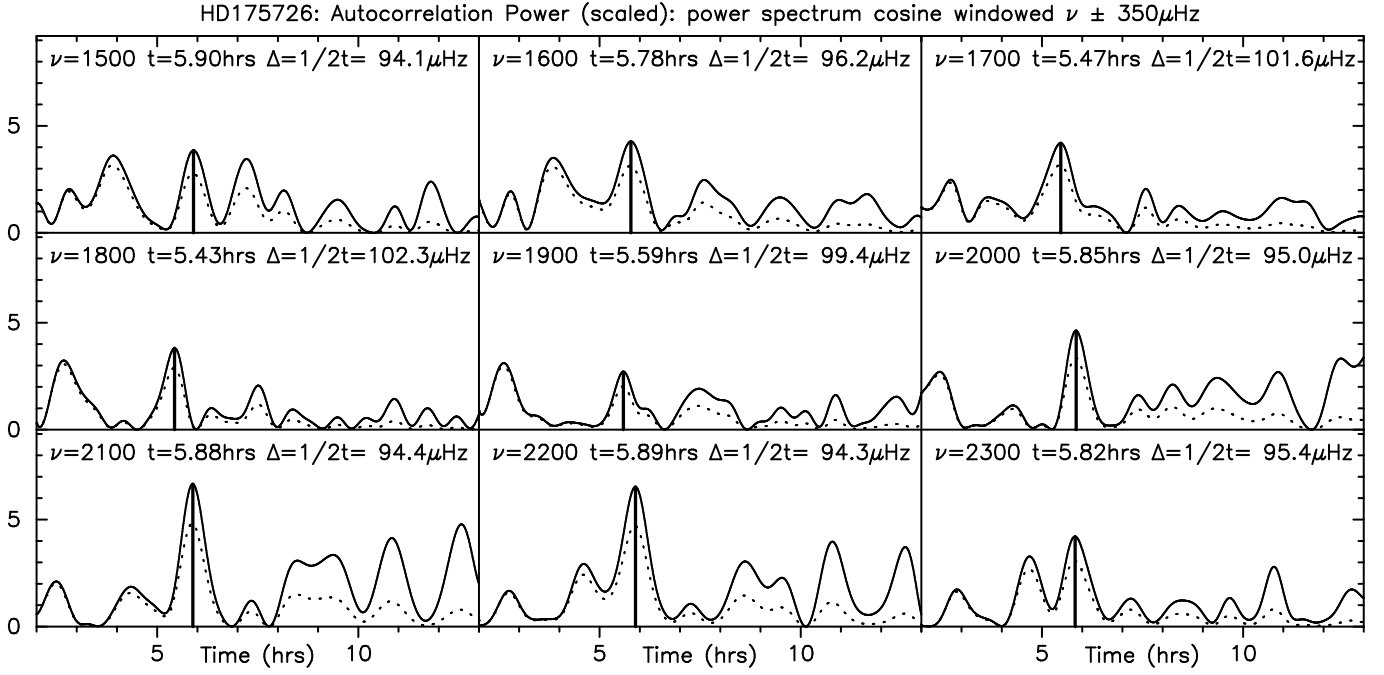


Fig. 6. Autocorrelation power for HD175726 cosine windowed with half width $350\mu\text{Hz}$ window. The dotted curves are obtained from the $15\mu\text{Hz}$ boxcar spectrum .

due to time resolution, window width and interference between signal and noise are considered in Mosser et al (2009). They also tested the reliability of the shape displayed in Fig 4 with an H0 test and concluded that the hypothesis that the signal is real is only rejected at the $\sim 1 - 10\%$ level.

As mentioned above one is here fighting against the noise - just how much is illustrated in Figure 6 which gives the autocorrelation power for a window of half width $350\mu\text{Hz}$ centred on frequencies in the range 1500 and $2300\mu\text{Hz}$. The peak around $98\mu\text{Hz}$ can just be seen - although how significant it is needs to be the subject of further investigation. Also shown in Fig 6 is the autocorrelation power using the boxcar spectrum shown in Fig 3b (dotted line).

4. Analysis and Modelling

In this section I use the angular frequency ω rather than the cyclic frequency ν ($\omega = 2\pi\nu$). I take the simplest possible model where the peaks in the frequency power spectrum are given by delta functions so

$$P(\omega) = \sum A_i \delta(\omega - \omega_i) \quad (1)$$

where ω_i are the eigenfrequencies. For simplicity of presentation I confine my attention to the case where the only modes are those with $\ell = 0, 1$ so that $\{\omega_i\} = \omega_{n,0}, \omega_{n,1}, \omega_{n+1,0}, \omega_{n+1,1}, \dots$

The eigenfrequencies satisfy the eigenfrequency equation (R&V 2000)

$$\omega_{n,\ell} = \Delta(n + \ell/2) + \frac{\Delta}{\pi}(\alpha(\omega) - \delta_\ell(\omega)), \quad \Delta = \frac{\pi}{T} \quad (2)$$

where α is the ℓ independent surface phase shift and δ_ℓ the ℓ dependent internal phase shift, T is a constant approximately the acoustic radius of the star. (Note any change in T can be absorbed into α)

Consider first the simple model consisting solely of a pair of adjacent modes $\omega_0 = \omega_{n,0}$ and $\omega_1 = \omega_{n,1}$, The Fourier transform

of the power spectrum consisting of these two modes is

$$F(t) = \int_0^\infty A_n [a_0 \delta(\omega - \omega_0) + a_1 \delta(\omega - \omega_1)] e^{i\omega t} dt \\ = A_n e^{i\omega_0 t} [a_0 + a_1 e^{i(\omega_1 - \omega_0)t}] \quad (3)$$

where A_n is an amplitude and the a_i visibility coefficients. In fact with just two modes a_1/a_0 is just the amplitude ratio of the modes.

The autocorrelation power (the power spectrum of the power spectrum) is $|F|^2$ which is then

$$A^2 = A_n^2 (a_0^2 + a_1^2 + 2a_0 a_1 \cos[(\omega_1 - \omega_0)t]) \quad (4)$$

This has its first peak (beyond zero) when $(\omega_1 - \omega_0)t = 2\pi$. If $\alpha, \delta_0 = \delta_1$ are all constant then from eqn(2) $\omega_1 - \omega_0 = \Delta/2$ and hence the first peak occurs at $t = 4T$. If the visibility coefficients are the same for a sets of pairs of modes then they all add up to give the same position of the first maximum (see R&V 2006).

However if, as is the real case, $\alpha, \delta_0, \delta_1$ vary with frequency and $\delta_1 \neq \delta_0$, the above pair analysis remains valid but $\omega_1 - \omega_0 \neq \Delta/2$. I therefore define the *half large separations*

$$\Delta_{10}(n) = \omega_{n,1} - \omega_{n,0}, \quad \Delta_{01}(n) = \omega_{n,0} - \omega_{n-1,1} \quad (5)$$

Their sum gives the ordinary large separations

$$\Delta_0(n) = \omega_{n,0} - \omega_{n-1,0} = \Delta_{01}(n) + \Delta_{10}(n-1) \quad (6a)$$

and difference the small separations (Roxburgh 1993, 2009)

$$d_{01}(n) = \omega_{n,0} - (\omega_{n,1} + \omega_{n-1,1})/2 = (\Delta_{01}(n) - \Delta_{10}(n-1))/2 \quad (6b)$$

The power from the pair of modes $\{\omega_{n,0}, \omega_{n,1}\}$ is

$$A^2 = A_n^2 (a_0^2 + a_1^2 + 2a_0 a_1 \cos[\Delta_{10}(n)t]) \quad (7)$$

which has its first maximum at $t = 2\pi/\Delta_{10}(n)$, whereas that from the pair $\{\omega_{n,1}, \omega_{n+1,0}\}$ is

$$A^2 = A_n^2 (a_0^2 + a_1^2 + 2a_0 a_1 \cos[\Delta_{01}(n+1)t]) \quad (8)$$

and has its first maximum at $2\pi/\Delta_{01}(n+1)$

As can be seen from Eqn (2)

$$\Delta_{10}(n) = \Delta \left(\frac{1}{2} + \frac{\alpha(\omega_{n,1}) - \alpha(\omega_{n,0})}{\pi} - \frac{\delta_1(\omega_{n,1}) - \delta_0(\omega_{n,0})}{\pi} \right) \quad (9)$$

$$\Delta_{0,1}(n+1) = \Delta \left(\frac{1}{2} + \frac{\alpha(\omega_{n+1,0}) - \alpha(\omega_{n,1})}{\pi} - \frac{\delta_0(\omega_{n+1,0}) - \delta_1(\omega_{n,1})}{\pi} \right) \quad (10)$$

so even were $\alpha, \delta_0, \delta_1$ constant, but $\delta_1 \neq \delta_0$, these differ by

$$\Delta_{01}(n+1) - \Delta_{10}(n) = \frac{2\Delta}{\pi}(\delta_1 - \delta_0) \quad (11)$$

For the Sun the difference $\Delta_{01} - \Delta_{10}$ varies with frequency ν from about $10\mu\text{Hz}$ at $\nu = 1000\mu\text{Hz}$ to $5\mu\text{Hz}$ at $\nu = 4000\mu\text{Hz}$.

For a large set of eigenfrequencies within a window (including $\ell = 2, 3$) the situation is more complicated since one cannot just add up the power in each pair but have to take the full Fourier transform. However one can see that if there is only one pair $\{\omega_{n,0}, \omega_{n,1}\}$ in a window, the position of the first maximum will be different from that with just the overlapping pair $\{\omega_{n,1}, \omega_{n+1,0}\}$. If there are a number of such sets within a window then one may expect the first peak in the autocorrelation power spectrum to be determined by the full large separations Δ_0, Δ_1 . Note that, at least for the Sun, the difference between Δ_0 and Δ_1 is much smaller than that between Δ_{01} and Δ_{10} since the inner phase shifts δ_0, δ_1 differ by much more than the change in α, δ_0 , and δ_1 between adjacent frequencies. Of course the actual windowed autocorrelation for a small set of frequencies depends on the amplitudes of all significant modes ($\ell = 0, 1, 2, 3$) within the window. For a theoretical model this could be calculated but for a real data set one can do no more than predict that for very narrow windows the first peak of the autocorrelation function will vary depending on whether it is an $\ell = 0, 1$ pair or an $\ell = 1, 0$ pair at the centre of the window, but for a wider window the peak is determined by a locally averaged large separation. This is the origin of the oscillatory behaviour of the $\pm 100\mu\text{Hz}$ windowed results for HD49933 displayed in Fig 4. It offers the possibility of determining the inner phase shift difference $\delta_1 - \delta_0$ as a function of frequency, which is an important diagnostic of the stellar interior and convective boundaries (Roxburgh 2009).

To demonstrate that this technique can, in principle, work I constructed a theoretician's ideal artificial noise free power spectrum by prescribing a surface phase shift $\alpha(\nu)$ and inner phase shifts $\delta_\ell(\nu)$, with the eigenfrequencies determined by the *Eigenfrequency equation* (R&V 2000)

$$2\pi\nu_{n,\ell}T = (n + \ell/2)\pi + \alpha(\nu) - \delta_\ell(\nu) \quad (12)$$

where in the classical limit $\delta_\ell(\nu) = \delta_0(\nu) + \ell(\ell + 1)D(\nu)$ and assumed a Lorentzian profile in the power spectrum of width $4\mu\text{Hz}$. The model power spectrum is shown in Fig 7a. I then applied the narrow windowed analysis with windows of $\pm 150, 300, 500\mu\text{Hz}$ which yielded the results displayed in Fig 7b. As expected the wider windows depart further from the actual large separations since the wider the window the greater the smoothing of the actual separations. The narrowest window shows the beginnings of a periodic modulation due to the difference between the *half large separations* Δ_{01} and Δ_{10} .

I then took very narrow windows of $\pm 60, 80, 100\mu\text{Hz}$ to test whether this can give the *half large separations* Δ_{01}, Δ_{10} defined above. Fig 7c shows the results for the full model power spectrum. The signal clearly shows these values but is polluted by the $\ell = 0, 2$ separations. Fig 7d was computed using the same model power spectrum but with only modes $\ell = 0, 1$. Here we successfully fit the *half large separations*.

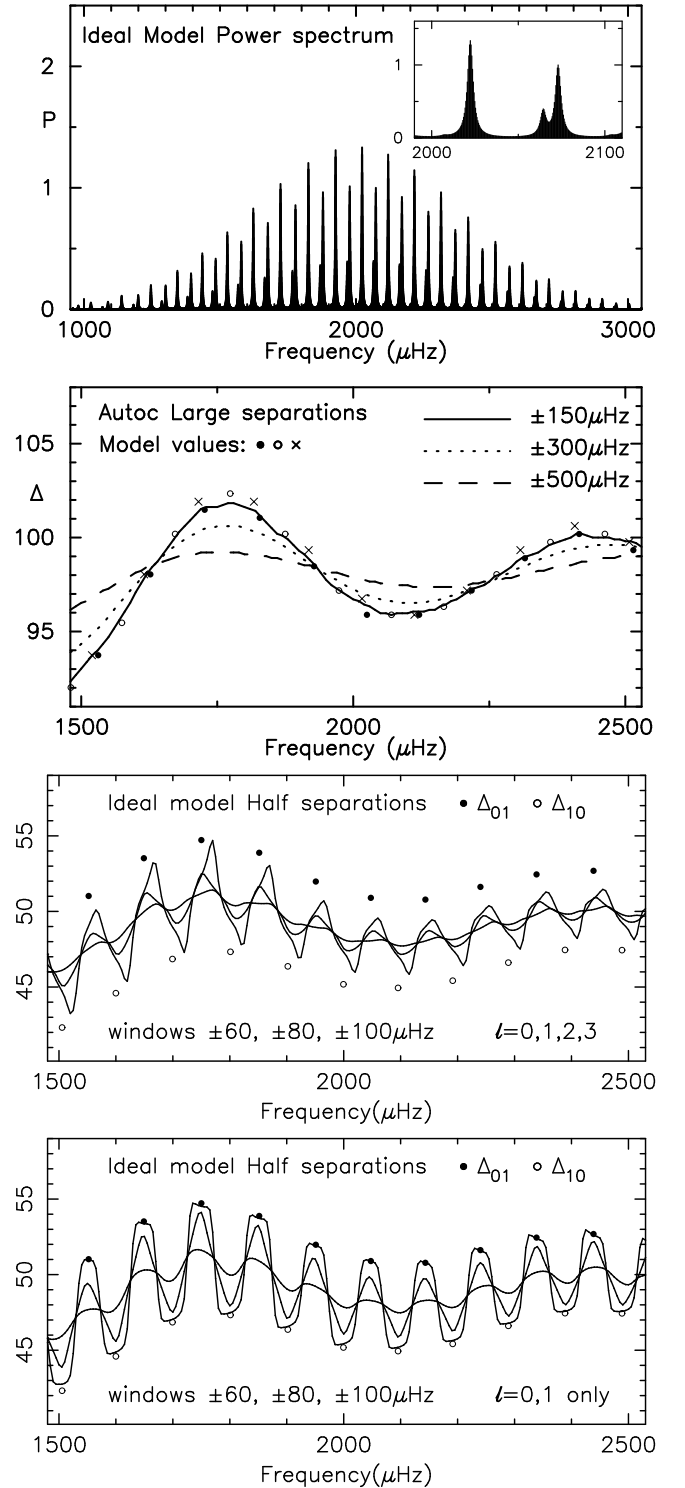


Fig. 7. a) Ideal model power spectrum; the inset shows an example of the line profiles. b) Large separation $\Delta(\nu)$ determined from windowed autocorrelations. The model values of Δ_ℓ are shown for $\ell = 0, 1, 2$. c) Half Large separation $\Delta_{01}(\nu), \Delta_{10}(\nu)$ determined from very narrow windowed autocorrelations of the full power spectrum with modes with $\ell = 0, 1, 2, 3$. The points are the values from the frequencies of the model. d) Half Large separation $\Delta_{01}(\nu), \Delta_{10}(\nu)$ determined from very narrow windowed autocorrelations of the power spectrum with only modes with $\ell = 0, 1$. The points are the values from the frequencies of the model.

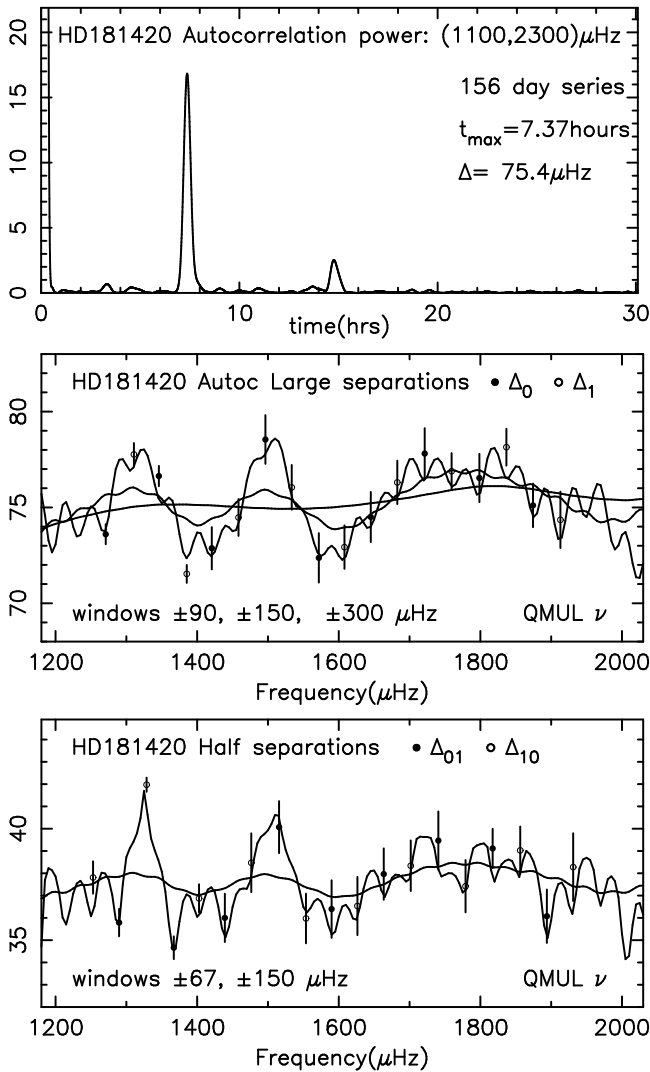


Fig. 8. a). Autocorrelation power for HD181420 with 1200μ Hz window. b). Autocorrelation Large separations for HD181420 with $\pm 90, 150, 300\mu$ Hz windows. c). Autocorrelation Half Large separations for HD181420 with $\pm 67, 150\mu$ Hz windows. The points are the values obtained from the frequencies

5. HD181420, HD181906

The other solar-type stars observed by CoRoT in the seismology field are HD181420. and HD181906. We here give the results of applying narrow windowed autocorrelations to these time series.

HD181420 (see Michel et al 2008, Barban et al 2009) is an F2 star which shows p-mode power in the range $1000-2500\mu$ Hz. I filtered the time series for low frequency variations, filled gaps and removed harmonics of the orbital period in the power spectrum. Fig 8a shows the autocorrelation power in a wide (1200μ Hz) window with a clean peak at $t = 7.37$ hrs corresponding to a mean large separation of 75.4μ Hz. I then used narrow windows of $\pm 90, 150, 300\mu$ Hz and determined the local large separations $\Delta(\nu)$, the results are in Fig 8b. The very narrow window shows large variations and the largest window a low amplitude smooth variation. Individual frequencies have been determined from the power spectrum within our group (see also Barban et al 2009) and the points in this figure are the large separations (with 1σ error estimates) determined from these frequencies; they agree remarkably well with the autocorrelation

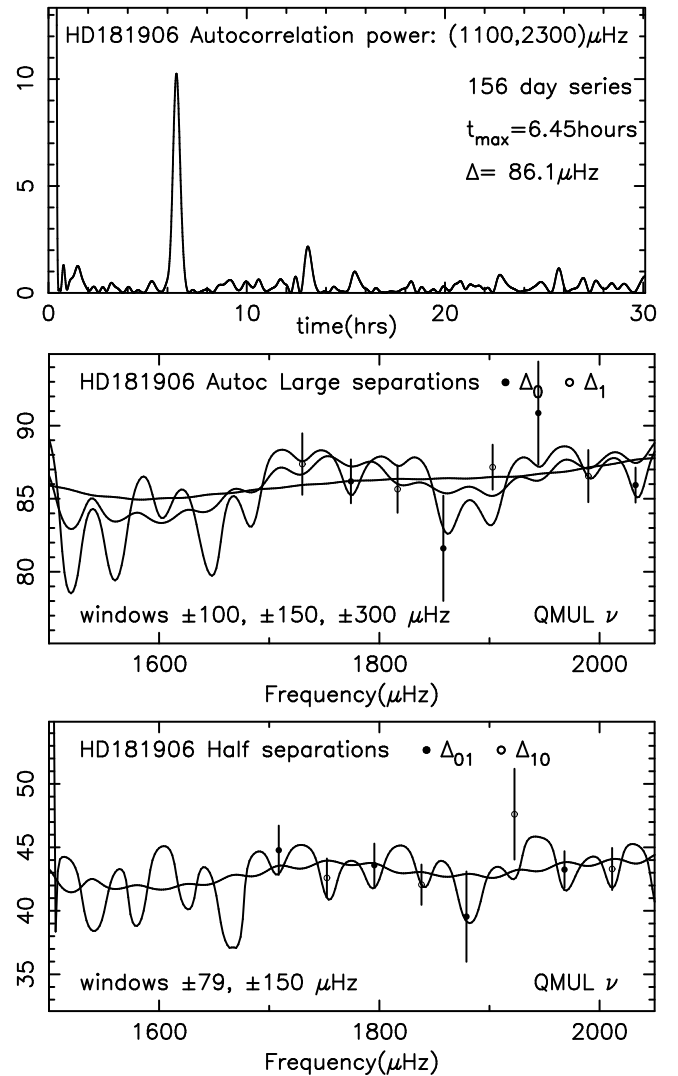


Fig. 9. a). Autocorrelation power for HD181906 with 1200μ Hz window. b). Autocorrelation Large separations for HD181906 with $\pm 100, 150, 300\mu$ Hz windows. c). Autocorrelation Half Large separations for HD181906 with $\pm 79, 150\mu$ Hz windows. The points are the values obtained from the frequencies.

estimate using the narrowest ($\pm 90\mu$ Hz) window. The frequencies obtained with interchanged $\ell = 0, 1$ mode identification (cf Scenario 1, Barban et al , 2009) do not fit so well.

I then took an even narrower window of $\pm 67\mu$ Hz to compare with the half separations Δ_{01}, Δ_{10} ; the results of are shown in Fig 8c compared with the half separations computed from the frequencies. Again this is very promising, we do not expect perfect agreement due to the contribution of the $\ell = 2$ modes to the windowed autocorrelation (see fig 7c). However the fit is better than with the alternative $\ell = 0, 1$ mode identification.

HD181906 is an F8 star observed by CoRoT that displays very low p-mode power in the frequency range $1000 - 2500\mu$ Hz (Michel et al 2008, Garcia e al 2009) and it is considerably fainter than HD49933 and HD 181420. With some difficulty estimates of frequencies have been determined within our group for this star although with some considerable uncertainties (see also Garcia et al 2009). I repeated the above analysis for this star and the results are given in Figures 9a, 9b and 9c. The autocorrelation has a clear peak at 6.45 hrs corresponding to a mean large separation of 86.1μ Hz. As can be seen from Figs 9b and 9c

the large and half-large separations derived from the estimated frequencies have very large estimated uncertainties but lie reasonable near to the curves derived from narrow frequency windowed autocorrelations. Further work needs to be done on both frequency extraction and the autocorrelations to improve these results.

6. Conclusions

The principal goal of this paper was to show that narrow frequency windowed autocorrelation can, in principle, reveal information on the variation of the large separation with frequency and therefore constitutes a tool that might be useful in obtaining some information about a star even when individual frequencies cannot be extracted. Much remains to be done to refine the technique: the theoretical analysis needs to be further developed and the nature of the interaction of noise with the autocorrelation power better understood. However the analysis presented here suggests that narrowed windows autocorrelations can in principle yield the variation with frequency of the large separations $\Delta(\nu)$ and that very narrow windows can yield the half large separations Δ_{01} and Δ_{10} and therefore give information on the internal phaseshift difference $\delta_1 - \delta_0$ which is a diagnostic of the internal structure of the star. These issues will be explored in a subsequent communication.

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References

- Appourchaux T, Michel E, Auvergne M, et al. 2008, A&A, 488, 705-714
 Barban C, Deheuvels S, Baudin F, et al, 2009. A&A this volume
 Brodskii M A, Vorontsov S V, 1989. Astron Zh, 15, 61-69. (English version: Sov Ast Lett 1989. 15, 27-30)
 Gabriel M, Grec G, Renaud C, et al., 1998, A&A, 338, 1109-1117.
 Garcia R, Regulo C, Samadi R, et al, 2009. A&A this volume
 Michel, E, Baglin, A Auvergne M, et al, 2008. Science, 322, 558-560.
 Mosser B, Roxburgh I W, Michel E, et al, 2009, A&A, this volume
 Roxburgh I W, 1993. in PRISMA, Report of Phase A Study, Appourchaux, T. et al., ESA SCI(93), p31-32.
 Roxburgh I W, 2008. private communication to CoRoT DAT team.
 Roxburgh I W, 2009. A&A, 493, 185-191.
 Roxburgh I W, Vorontsov S V, 1994. MNRAS, 267, 297-302
 Roxburgh I W, Vorontsov S V, 2000. MNRAS, 317, 141-150.
 Roxburgh I W, Vorontsov S V, 2006. MNRAS, 369, 1491-1496.
 Vorontsov S V, Zharkov V N, 1988. Itogi Nauki Tek, Ser Astro, 38, 253-338 (English version: Astro & Space Phys Rev, 1989, 7, 1-87)