R W O C
D S P : α C A & HD49933

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for First CoRoT International Symposium

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Abstract

We study the effect of using different observed quantities (oscillation frequencies, binarity, interferometric) and the impact of their accuracy on constraining the uncertainities of global free stellar parameters (i.e. the mass, the age etc.). We use the Singular Value Decomposition (SVD) formalism to analyse the behavior of the χ^2 fitting function around its minimum. We apply this tool to α Cen Afor which, seismic, binarity and interferometric properties are known with high accuracy. We also apply this tool the study of the CoRoT target HD49933 for which mass and radius constraints are not available.This method relates the errors in observed quantities to the precision in the model parameters. We determine how changes of the accuracy of observable constraints affect the precision obtained on the global stellar parameters for relatively distant systems.

Model Description

The stellar models of α Cen A Cen A and HD49933 are computed with the stellar evolution code CESAM2k (Morel 1997) starting from the ZAMS.The adopted physical description for convection calculation is the standard MLT (Böhm- Vitense 1958) for α Cen A and the FST (Canuto & Mazzitelli 1996) for HD49933; the OPAL opacities (Iglesias & Roger 1996) completed at low temperatures with the opacities of Alexander & Ferguson (1994); the OPAL equation of state; and an Eddington atmosphere as the surface boundary condition. The adiabatic oscillation frequencies are calculated for ℓ = 0-3 and $n = 15-25$ for α Cen A and $\ell = 1-2$ and $n = 13-27$ for HD49933 using the adiabatic oscillation code Losc (Scuflaire et al. 2007).

To construct **the derivative matrix** D, we vary each of the parameters. Each derivative is computed from differences centered on the reference parameter values given in Table 2. The interval δx has to be sufficiently small such that the linear approximation is good, yet still large enough to avoid numerical problems.

The increments (δx_j) used for the α Cen A and HD49933

δx_j	τ(Gyr)	α	M	Y	Z/X	d_{ov}
α Cen A	20	0.05	0.005	0.003	0.0005	×
HD49933	20	0.06	0.005	0.005	0.0004	0.08

The Method

Given a set of n measurements $y_{obs,i}$ (e.g. T_{eff}, L, Δ , etc.) with associated error bars and a set of m free parameters x_j (e.g. τ, α, M, etc.), we first determine the reference model (RM) which minimizes the χ^2 fitting function defined as:

$$\chi^2 = \sum_i^N \frac{(y_{obs,i} - y_{the,i})^2}{\sigma_i^2} \quad (1)$$

T χ^2 SVD

$$\chi^2 = ||\mathbf{y_{obs}} - \mathbf{y_{the}}||^2$$

$$\mathbf{y_{the}}(\mathbf{x_0} + \delta\mathbf{x}) = \mathbf{y_{the}}(\mathbf{x}) + \mathbf{D}.\delta\mathbf{x}$$

$$\Delta\chi^2 = ||\mathbf{D}\delta\mathbf{x}||^2$$

$$\mathbf{D}_{n\times m} = \frac{1}{\sigma_i}\frac{\partial y_{the,i}}{\partial x_j}$$

SVD → $D_{n\times m} = U_{n\times m} W_{m\times m} V^T_{m\times m}$

$\sum_{i=1}^N U_{ik}U_{in} = \delta_{kn}$ W is an diagonal matrix $\sum_{j=1}^N V_{jk}V_{jn} = \delta_{kn}$

T χ^2 -

m-D .T

V

, $\mathbf{W}^{-1}$

.

$$\Delta\chi^2 = \frac{||V^{(1)}\delta x||^2}{W_1^{-2}} + \ldots + \frac{||V^{(m)}\delta x||^2}{W_m^{-2}} \quad (2)$$

E m

n .

$$Cov(\delta x_j, \delta x_k) = \sum_i^N \frac{V_{ji}V_{ki}}{W_i^2} \quad (3)$$

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Application to Different Astrophysical Situation Based on the α Cen A

The list of observations including their standard errors, defining our RM, is given Table 1. The characteristics of our RM are obtained using the Levenberg-Marquardt algoritm that searches the best-fit parameters by χ^2 minimisations.

Observations of α Cen A and the properties of the RM

	T_{eff}[7]	$L/L_\odot$[7]	Z/X_0[11]	$R/R_\odot$[9]	Δ[4]	δ_{02}[4]	$M/M_\odot$[5]
α Cen A	5810	1.522	0.039	1.224	105.5	5.6	1.105
The errors(σ)	50K	0.030	0.06	0.003	0.1μHz	0.7μHz	0.007
RM	5782	1.516	0.039	1.229	105.5	5.7	1.099
Parameters of RM		$\tau(Gyr)$	α	M	Y_0	Z/X_0	
		5.65	1.6747	1.099	0.280	0.039	

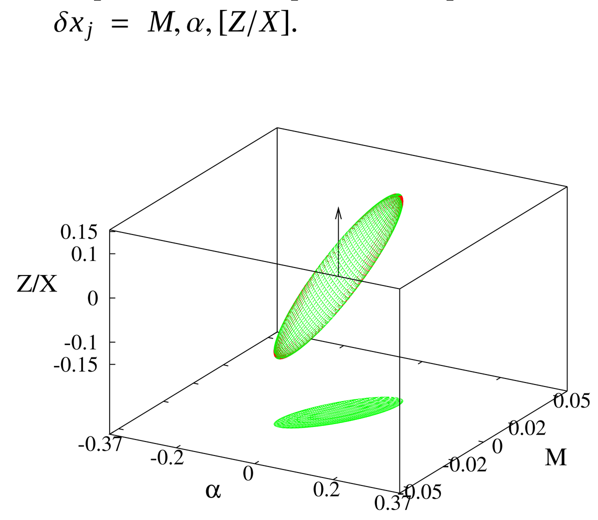
We study several different cases to cover a large range of realistic situations. Case 1 describes α Cen A (d = 1.3pc), Case 2 corresponds to a system located ten times further away (d = 13pc). For distant objects where the binary and interferometric data are unavailable, the seismic data are the major source of information (Case 3). We discuss also the influence of the seismic data precision(Case 4). Table 2. shows the results from these different cases.

F 1: The rms error on the mass ($\epsilon(M)$), taking into account different set of observables for α Cen A and for the system if it were located at 13pc. The set of classical observable $Q_i = (T_{eff}, L/L_\odot, Z/X_0)$ are included in all cases. The symbol (√) indicates that the observable is included in the SVD analysis, if not ×. The set of parameter is $P = \alpha, M, [Z/X]$

Observables					d=1.3pc	d=13pc
Classic		Seismic (μHz)				
$R/R_\odot$	$M/M_\odot$	(Δ, δ_{02})	$(\Delta_i, \delta_{02,i})$	σ_Δ	$\epsilon(M)$ (%)	$\epsilon(M)$(%)
×	×	×	×	–	2.21	2.26
×	×	√	×	0.1	2.09	2.19
×	×	×	√	–	1.73	1.75
×	×	√	×	2	2.12	2.20
√	×	×	×	–	2.20	2.24
√	×	√	×	0.1	0.75	2.13
√	×	×	√	–	0.75	1.72
√	×	√	×	2	1.93	2.15
×	√	×	×	–	0.61	2.12
×	√	√	×	0.1	0.61	2.06
×	√	×	√	–	0.60	1.68
×	√	√	×	2	0.61	2.07
√	√	×	×	–	0.61	2.10
√	√	√	×	0.1	0.48	2.01
√	√	×	√	–	0.48	1.66
√	√	√	×	2	0.60	2.03

Note: The oscillation data in this poster is expressed by the mean (Δ, δ) and individual (Δ_i, δ_i) separation. $\epsilon(M) = Var\ \frac{dM}{M})^{1/2}$

F 2: A sketch of the error ellipsoid in 3-D parameter space $\delta x_j = M, \alpha, [Z/X]$.



In all these cases, the axes of the error ellipsoids corresponding to the largest singular value has an important contribution from the mass and less from other parameters ($W_M^1 \approx 4100 >> W_\alpha^2 \approx 47 > W_{Z/X}^3 \approx 18$). The mass is the best constrained parameter with these observables. We therefore focus here on its uncertainty.

Results

- **Case 1** The seismic constraint on the mean large separation Δ and the interferometric constraint on the radius $R/R_\odot$ give about the same precision on the M parameter (ϵ = 2.09%, and 2.20% ,respectively). If If both are considered together, $\epsilon(M) \searrow$ 0.75%. This comes from the fact that $M \propto \Delta^2R^3$. **Using the mean Δ or the individual Δ_i large separation give about the same precision on the mass, as we are close to the asymptotic regime**.
- **Case 2** Comparing Case 2 with Case 1 allows to estimate the effect of increasing distance on determination of the precision of mass parameter for the same combination of observables. The observables depending on the distance (i.e, $M/M_\odot$, $R/R_\odot$, $L/L_\odot$) become less effective to constrain M. For example $\epsilon(M) \nearrow$ from ϵ = 0.75% to ϵ = 2.13% when the $R/R_\odot$ and the (Δ, δ) are considered together.
- **Case 3** The seismic information alone gives ϵ = 2.19%. As the precision on the seismic data does not depend on distance, the ϵ does not change (ϵ = 2.09% for d=1.3pc, ϵ = 2.19% for d=13pc). Case 3 vs. Case 1, even if the $M/M_\odot$ or $R/R_\odot$ were available they would not $\searrow$ significantly $\epsilon(M)$ (ϵ = 2.13% if the $R/R_\odot$ is available, ϵ = 2.06% if the $M/M_\odot$).
- **Case 4** We $\searrow$ $\sigma_{\Delta\nu}$ = 2 μHz as the worst case scenario. $\epsilon(M) \nearrow$ **only slightly because of the flatness of the error ellipsoid**.

Application to the HD49933

The list of observations including their standard errors, defining our RM, is given Table 2. We estimate here the arithmetic mean small separation $\delta_{01} = -0.48 \pm 0.32$ derived from the frequecy difference $\delta_{01} = \nu_{n,0} - (\nu_{n,1} + \nu_{n-1,1})/2$ from ℓ = 0 and 1 modes.

Observations of HD49933 and the properties of the RM

	T_{eff}[4]	$L/L_\odot$[2]	Z/X_0 [14]	$R/R_\odot$	Δ[2]	δ_{01}	$M/M_\odot$
HD49933	6780	0.53	0.01024	–	85.9	-0.48	–
The errors(σ)	130K	0.01	0.0071	–	0.15 μHz	0.32 μHz	–
RM	6669	0.54			85.63		
Parameters of RM		$\tau(Gyr)$	α	M	Y_0	Z/X_0	d_{ov}
		3724.903	1.031	1.1544	0.282	0.1129	0.4016

We consider 3 different scenarios which reflect various possibilities for mode frequency identification of HD49933. Bearing in mind the lack of proper mode identifiaction for l=0 and l=2, in (Case 1) we consider when the small frequency information δ is not available. Case 2 shows the addition of small frequency information (the mean δ_{01} and individual $\delta_{01,i}$ small separation). In Case 3, we reduce the error of the seismic data by a factor 10 for gain a maximum advantage of the increased accuracy of seismology.

F 3: The rms errors on the different sets of parameters (P_1, P_2, P_3) of the mass M, the age τ, the mixing length parameter α, and overshooting ov for HD49933. We keep the other parameters fixed at their correct values. The set of classical observable $Q_i = T_{eff}, L/L_\odot$ is included in all cases.

		Case 1		Case 2		Case 3	
	ϵ (P) %	Δ	Δ_i	Δ, δ_{01}	$\Delta_i, \delta_{01,i}$	$\Delta, \delta_{01}(\sigma_{\Delta,\delta}\searrow 10)$	$\Delta_i, \delta_{01,i}(\sigma_{\Delta_i,\delta_{01,i}}\searrow 10)$
P_1	M	45.88	0.47	0.83	0.45	0.56	0.08
	α	541.73	3.44	7.43	1.93	1.95	0.36
	τ	436.04	1.88	6.3	1.87	2.50	0.35
P_2	M	4.83	0.36	0.28	0.12	0.27	0.01
	α	177.51	3.31	6.48	2.81	5.91	0.31
	d_{ov}	617.61	9.20	9.61	1.83	3.81	0.19
P_3	M	15.20	0.46	0.69	0.32	0.68	0.04
	τ	212.28	6.82	3.85	1.55	3.52	0.18
	d_{ov}	918.50	34.56	6.74	1.04	1.77	0.11

Note: The oscillation data in this poster is expressed by the mean (Δ, δ) and individual (Δ_i, δ_i) separation. $\epsilon(P) = Var\left(\frac{dP}{P}\right)^{1/2}$

Results

- **Case 1** In the first column, we obtain very high incertitues for the P_1 set of parameters because of the strong correlation between the parameters: $(M-\alpha)$, $(M-\tau)$, and particularly $(\alpha-\tau)$. To explain this case, we show the parameter matrix **V** as well as their corresponding singular values obtained by the SVD analysis.

$$\mathbf{V} = \begin{pmatrix} \tau & 0.178215423 & -0.759472712 & 0.62565203 \\ \alpha & -0.0602659768 & 0.626212513 & 0.77731969 \\ M & 0.982144223 & 0.176235888 & -0.065830363 \\ & \downarrow & \downarrow & \downarrow \\ & W_1 = 2602.738 & W_2 = 32.191 & W_3 = 0.143 \end{pmatrix}$$

The smallest singular value W_3 is assosiated with the third column of **V** which principally contributes to α. It is worst determined. Hence, the axes of the error ellipsoid corresponding to α degenerate in direction of the α parameter. Because of the obliqueness of the error ellipsoid, consequently, the precision obtained on the M and τ is very low.

Besides, using Δ_i makes substantial improvements in the precision of all parameters. In the absence of the δ separation data, the error on the parameter d_{ov} is very high.

- **Case 2** Providing complete mode frequency information has relatively large effect on the determination of uncertainities of all stellar parameters. One obtains remarkable $\searrow$ on parameter errors.
- **Case 3** We show the importance of the very precise seismic data. Assuming that one gets better precison frequency thanks to future works on high accuracy mode identification, here the accuracy of sismic data is $\nearrow$ by a factor of 10. **In this case, one can obtain a precision on the mass parameter better than $\epsilon(M) < 1$%**. It is also interesting that the mean δ separation has a strong impact on the precision of overshooting.

Conclusion & Discussion

- Comparision α Cen A with HD49933 , using the individual (Δ_i, δ_i) separation instead of the mean (Δ, δ) large separation in the case HD49933 makes substantial improvements in the precision of all parameters **since we are not in the asymptotic region**.
- The changes on the sismic precision of HD49933 are not lead to overconstrained the stellar models due to having a high accuracy on classical observations. On the other hand in the case HD49933 we have only several classical data ($T_{eff}, L/L_\odot$), and due to having the large error on T_{eff}, they are insuffecient to constrain the stellar parameters. As a consequence, the addition of sismic data and the changes in their precision espcially if one has individual not just the mean separations yield a tremendous gain.
- **The better the sismic information is precise, the better the stellar parameters constrain.**

Symposium CNES-CoRoT

Maison International, Paris-France, February 2–5, 2009