Mode identification in rapidly rotating stars

D. R. Reese\textsuperscript{1}, M. J. Thompson\textsuperscript{1}, K. B. MacGregor\textsuperscript{2}, S. Jackson\textsuperscript{2}, A. Skumanich\textsuperscript{2}, T. S. Metcalfe\textsuperscript{2}

\textsuperscript{1}University of Sheffield, \\
\textsuperscript{2}High Altitude Observatory, Boulder

February 3, 2008
many intermediate and massive stars rotate rapidly
their structure and evolution remain poorly understood

Monnier et al. (2007)

CoRoT targets

<table>
<thead>
<tr>
<th>HD</th>
<th>Type</th>
<th>$v \cdot \sin i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>181555</td>
<td>δ Scuti</td>
<td>170$^a$</td>
</tr>
<tr>
<td>49434</td>
<td>γ Dor</td>
<td>90$^a$</td>
</tr>
<tr>
<td>171834</td>
<td>γ Dor</td>
<td>72$^a$</td>
</tr>
<tr>
<td>170782</td>
<td>δ Scuti</td>
<td>198$^a$</td>
</tr>
<tr>
<td>170699</td>
<td>δ Scuti</td>
<td>&gt; 200$^a$</td>
</tr>
<tr>
<td>177206</td>
<td>δ Scuti</td>
<td>&gt; 200$^b$</td>
</tr>
</tbody>
</table>

$^a$Poretti et al. (2005), $^b$Corotsky:
Recent 2D models

- Maeder & Meynet et al. (1997-2001)
- Self-Consistent Field (SCF) method: Jackson et al. (2004, 2005), MacGregor et al. (2007)
Recent 2D models

- Maeder & Meynet et al. (1997-2001)
- Self-Consistent Field (SCF) method: Jackson et al. (2004, 2005), MacGregor et al. (2007)

Recent 2D calculations of acoustic modes

- Espinosa et al. (2004)
- Lovekin et al. (2008, 2009)
- Lignières & Georgeot (2008, submitted to A&A)
Important results on the acoustic spectrum

- mode classification into regular and chaotic classes
  - polytropic case, using both ray dynamics and a normal mode analysis (Lignières & Georgeot, 2008, submitted to A&A)
  - extended to SCF models based on a normal mode analysis (Reese et al., submitted to A&A)
Island modes

- rotating counterparts to modes with low $\ell - |m|$ values
- the most visible of the regular modes
- new set of quantum numbers ($\tilde{n}, \tilde{\ell}, m$)
- asymptotic formula valid for low azimuthal orders:

$$\omega = \tilde{n}\Delta\tilde{n} + \tilde{\ell}\Delta\tilde{\ell} + m^2\Delta\tilde{m} - m\Omega + \tilde{\alpha}$$
Island modes

- Rotating counterparts to modes with low $\ell - |m|$ values
- The most visible of the regular modes
- New set of quantum numbers ($\tilde{n}$, $\tilde{\ell}$, $m$)
- Asymptotic formula valid for low azimuthal orders:

$$\omega = \tilde{n}\Delta\tilde{n} + \tilde{\ell}\Delta\tilde{\ell} + m^2\Delta\tilde{m} - m\Omega + \tilde{\alpha}$$

- Can this be used to identify pulsation modes?
A new mode identification scheme

\[ n_0 \leq \tilde{n} \leq n_1 \]
\[ l_0 \leq \tilde{l} \leq l_1 \]
\[ -m_0 \leq m \leq m_0 \]

\[ \Delta_n, \; \Delta_l, \; \Delta_m, \; \Omega, \; \alpha \]
A new mode identification scheme

\[ \omega = \tilde{n}\Delta_{\tilde{n}} + \tilde{\ell}\Delta_{\tilde{\ell}} + m^2\Delta_{\tilde{m}} - m\Omega + \tilde{\alpha} \]
A new mode identification scheme

\[ \omega = \tilde{n} \Delta \tilde{n} + \tilde{\ell} \Delta \tilde{\ell} + m^2 \Delta \tilde{m} - m\Omega + \tilde{\alpha} \]
A new mode identification scheme

Scientific context

Island modes

A new mode identification scheme

Results

Conclusion

A new mode identification scheme

\[ \omega = \tilde{n} \Delta_{\tilde{n}} + \tilde{\ell} \Delta_{\tilde{\ell}} + m^2 \Delta_{\tilde{m}} - m\Omega + \tilde{\alpha} \]
A new mode identification scheme

\[ \omega = \tilde{n}\Delta_{\tilde{n}} + \tilde{\ell}\Delta_{\tilde{\ell}} + m^2\Delta_{\tilde{m}} - m\Omega + \tilde{\alpha} \]
A new mode identification scheme

\[ \omega = \tilde{n}\Delta_{\tilde{n}} + \tilde{\ell}\Delta_{\tilde{\ell}} + m^2\Delta_{\tilde{m}} - m\Omega + \tilde{\alpha} \]

- this yields a mode identification
A new mode identification scheme

\[ \omega = \tilde{n}\Delta_{\tilde{n}} + \tilde{\ell}\Delta_{\tilde{\ell}} + m^2\Delta_{\tilde{m}} - m\Omega + \tilde{\alpha} \]

- this yields a mode identification
- this can then be used to readjust \( \Delta_{\tilde{n}}, \Delta_{\tilde{\ell}}, \Delta_{\tilde{m}}, \alpha, \Omega \)
A new mode identification scheme

\[ \omega = \tilde{n} \Delta \tilde{n} + \tilde{\ell} \Delta \tilde{\ell} + m^2 \Delta \tilde{m} - m\Omega + \tilde{\alpha} \]

- this yields a mode identification
- this can then be used to readjust \( \Delta \tilde{n}, \Delta \tilde{\ell}, \Delta \tilde{m}, \alpha, \Omega \)
- adjust parameters and obtain a closer fit to observed frequencies
General procedure

- the previous slide explained how to find a plausible mode identification for a given set of parameters
- repeat the same procedure while scanning a 5D parameter space
- a slightly different set of parameters is used:

$$\Delta \tilde{n}, \quad \frac{\Delta \tilde{\ell}}{\Delta \tilde{n}}, \quad \frac{\Delta \tilde{m}}{\Delta \tilde{n}}, \quad \frac{\alpha}{\Delta \tilde{n}}, \quad \frac{\Omega}{\Delta \tilde{n}}$$

- the synthetic spectrum does not need to be sorted again when $\Delta \tilde{n}$ or $\frac{\alpha}{\Delta \tilde{n}}$ change value
General procedure

- the previous slide explained how to find a plausible mode identification for a given set of parameters
- repeat the same procedure while scanning a 5D parameter space
- a slightly different set of parameters is used:

\[ \Delta \tilde{n}, \quad \frac{\Delta \tilde{l}}{\Delta \tilde{n}}, \quad \frac{\Delta \tilde{m}}{\Delta \tilde{n}}, \quad \frac{\alpha}{\Delta \tilde{n}}, \quad \frac{\Omega}{\Delta \tilde{n}} \]

- the synthetic spectrum does not need to be sorted again when \( \Delta \tilde{n} \) or \( \frac{\alpha}{\Delta \tilde{n}} \) change value

Computational cost

- about 1 hour for \( 50^5 \) parameter combinations
Synthetic "Observations"
Reese, Thompson, MacGregor, Jackson, Skumanich, Metcalfe

Mode identification in rapidly rotating stars
Synthetic "Observations"
Reese, Thompson, MacGregor, Jackson, Skumanich, Metcalfe

Mode identification in rapidly rotating stars
avoided crossings cause deviations between asymptotic formula and numerical frequencies
avoided crossings cause deviations between asymptotic formula and numerical frequencies

• these deviations become much smaller at high radial orders
Reese, Thompson, MacGregor, Jackson, Skumanich, Metcalfe

Mode identification in rapidly rotating stars
the previous example all contained 50 observed modes
what happens when you change the number of modes?
- the previous example all contained 50 observed modes
- what happens when you change the number of modes?
Scientific context

Island modes

A new mode identification scheme

Results

Conclusion

Chaotic modes are also likely to be visible in the frequency spectrum
what happens to mode identification?
what happens to mode identification?

- low radial orders
- high radial orders
1 random frequency

Reese, Thompson, MacGregor, Jackson, Skumanich, Metcalfe

Mode identification in rapidly rotating stars
3 random frequencies

Reese, Thompson, MacGregor, Jackson, Skumanich, Metcalfe

Mode identification in rapidly rotating stars
Improving the mode identification scheme

- allow the algorithm to reject the $N_{\text{cut}}$ worst solutions
- keeping all frequencies
- rejecting 20 frequencies
Conclusion

- it is possible to identify acoustic modes in rapidly rotating stars based on the asymptotic formula for island modes
  - a sufficient number of modes is needed
  - modes with high radial orders yield better results
  - chaotic modes are problematic
- possible improvements:
  - correlate theoretical mode visibilities with observed mode amplitudes
  - use results from 2D pulsation calculations to restrict parameter space
  - take non-adiabatic effects into account to know which modes are excited
- direct comparison between observations and numerical calculations
Mode identification in rapidly rotating stars

Reese, Thompson, MacGregor, Jackson, Skumanich, Metcalfe
Mode identification in rapidly rotating stars

Reese, Thompson, MacGregor, Jackson, Skumanich, Metcalfe
Mode identification in rapidly rotating stars

Reese, Thompson, MacGregor, Jackson, Skumanich, Metcalfe

Scientific context

Island modes

A new mode identification scheme

Results

Conclusion

$N_{\text{cut}} = 20$, $N_{\text{rand}} = 0$
Scientific context

Island modes

A new mode identification scheme

Results

Conclusion

$N_{\text{cut}} = 20$, $N_{\text{rand}} = 20$