Mode identification in rapidly rotating stars

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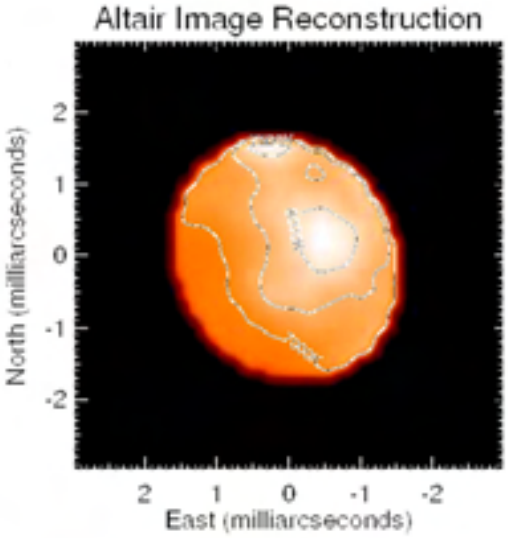
Scientific context

- many intermediate and massive stars rotate rapidly
- their structure and evolution remain poorly understood



Monnier et al. (2007)

CoRoT targets

HD	Type	$v \cdot \sin i$
181555	δ Scuti	170^a
49434	γ Dor	90^a
171834	γ Dor	72^a
170782	δ Scuti	198^a
170699	δ Scuti	$> 200^a$
177206	δ Scuti	$> 200^b$

[a] Poretti et al. (2005), [b] Corotsky: `http://smsc.cnes.fr/COROT/A_corotsky.htm`.

Recent 2D models

- Maeder & Meynet et al. (1997-2001)
- Roxburgh (2004, 2006)
- Self-Consistent Field (SCF) method: Jackson et al. (2004, 2005), MacGregor et al. (2007)
- ESTER project: Rieutord (2006), Espinosa & Rieutord (2007)

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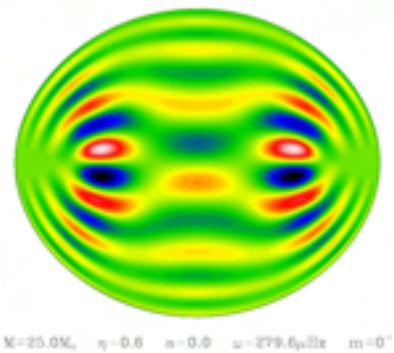
Recent 2D calculations of acoustic modes

- Espinosa et al. (2004)
- Lovekin et al. (2008, 2009)
- Lignières et al. (2006), Reese et al. (2006, 2008, submitted to A&A)
- Lignières & Georgeot (2008, submitted to A&A)

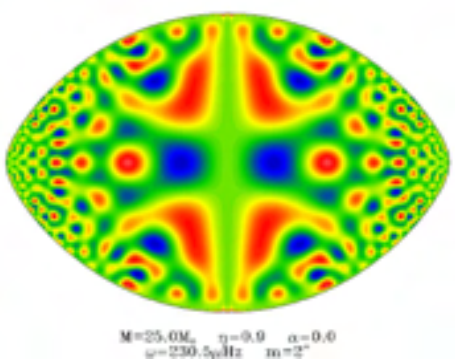
Important results on the acoustic spectrum

- mode classification into regular and chaotic classes
  - polytropic case, using both ray dynamics and a normal mode analysis (Lignières & Georgeot, 2008, submitted to A&A)
  - extended to SCF models based on a normal mode analysis (Reese et al., submitted to A&A)

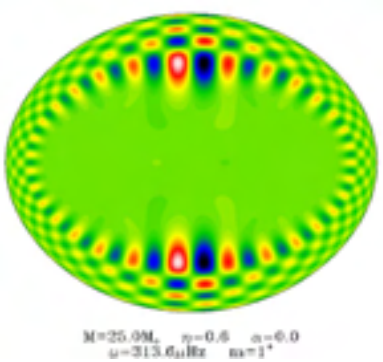
Island



Chaotic



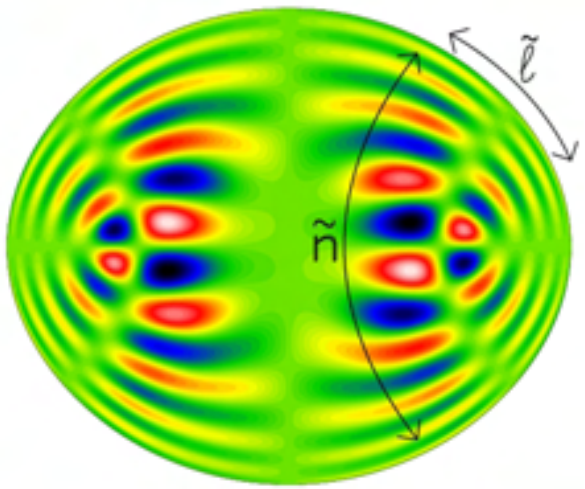
Whispering gallery



Island modes

- rotating counterparts to modes with low $\ell - |m|$ values
- the most visible of the regular modes
- new set of quantum numbers $(\tilde{n}, \tilde{\ell}, m)$
- asymptotic formula valid for low azimuthal orders:

$$\omega = \tilde{n}\Delta_{\tilde{n}} + \tilde{\ell}\Delta_{\tilde{\ell}} + m^2\Delta_{\tilde{m}} - m\Omega + \tilde{\alpha}$$



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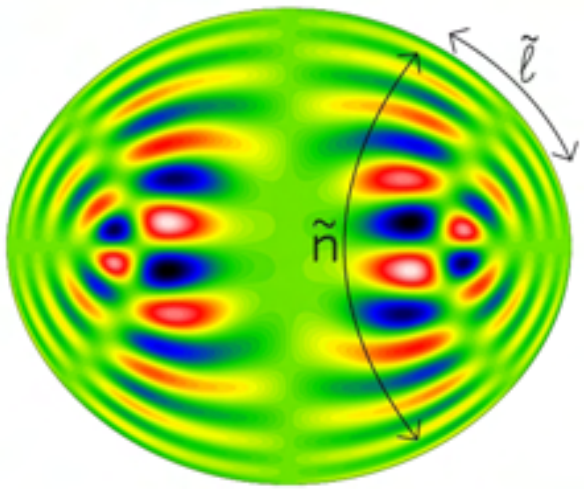
- can this be used to identify pulsation modes?

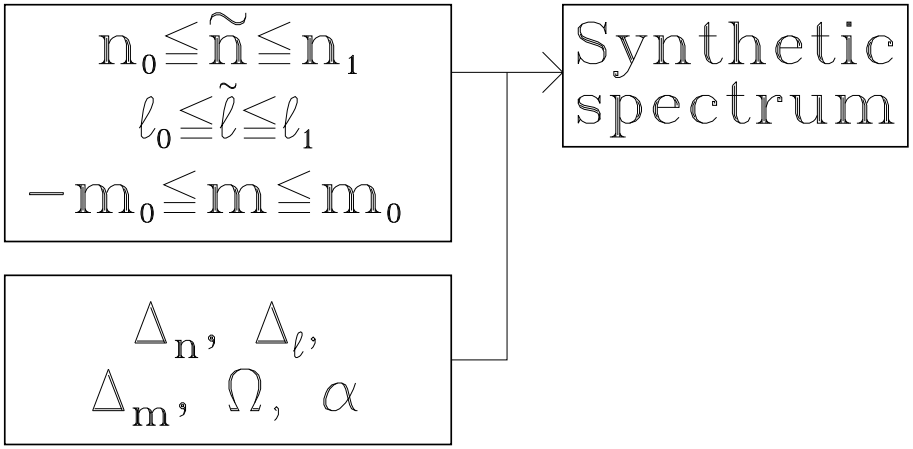


A new mode identification scheme

$$n_0 \leq \tilde{n} \leq n_1$$
$$\ell_0 \leq \tilde{\ell} \leq \ell_1$$
$$-m_0 \leq m \leq m_0$$

$$\Delta_n, \ \Delta_\ell, \ \Delta_m, \ \Omega, \ \alpha$$

A new mode identification scheme



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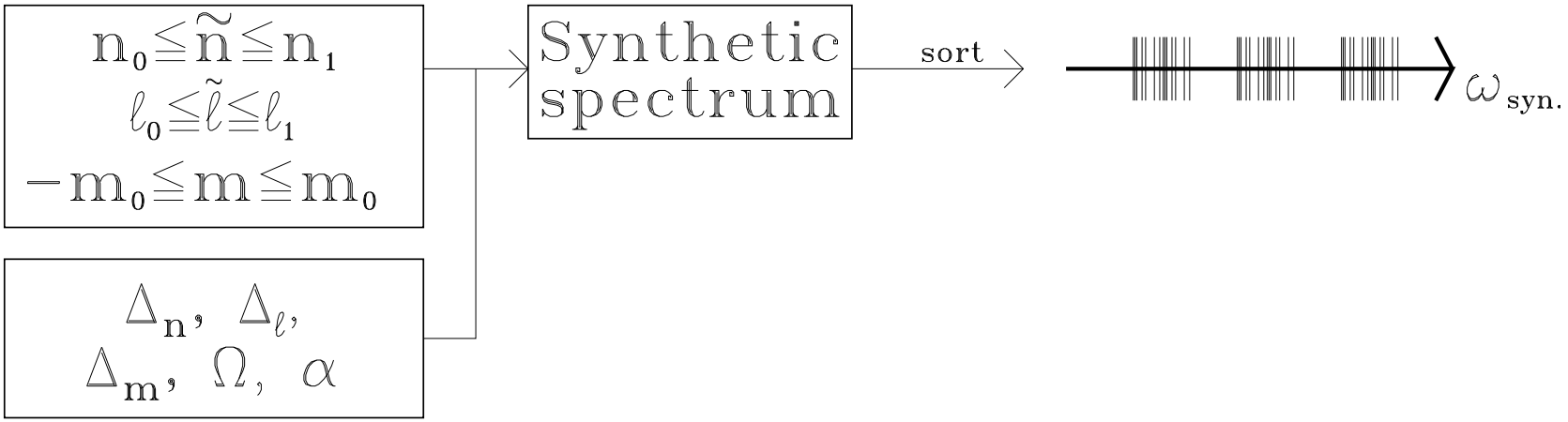

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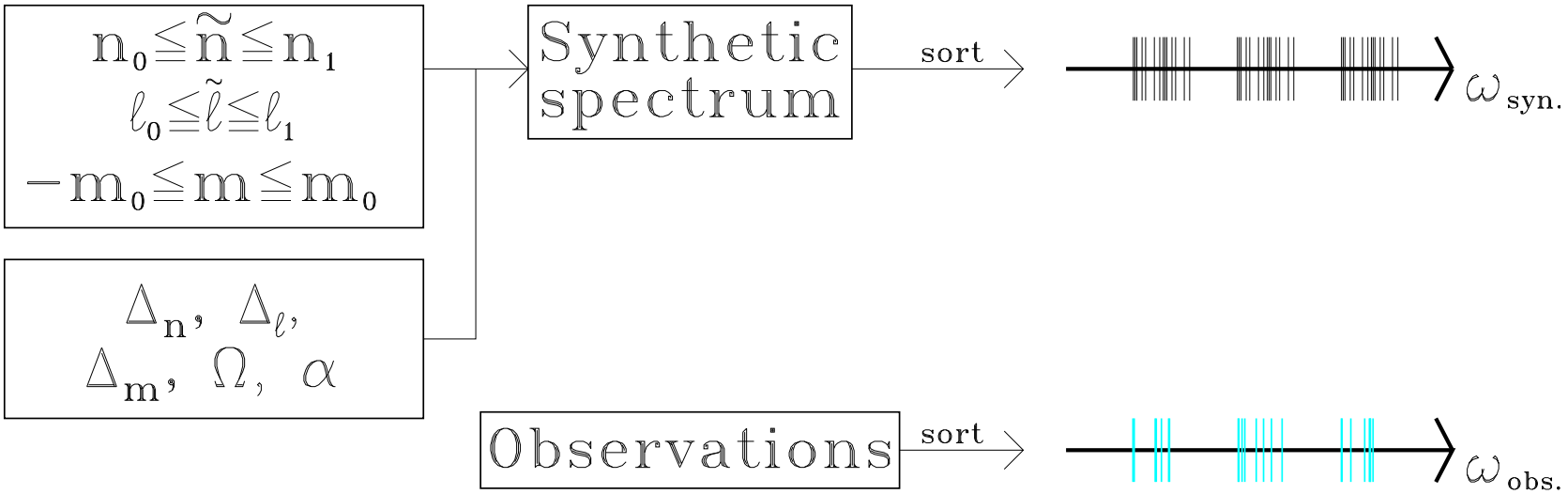

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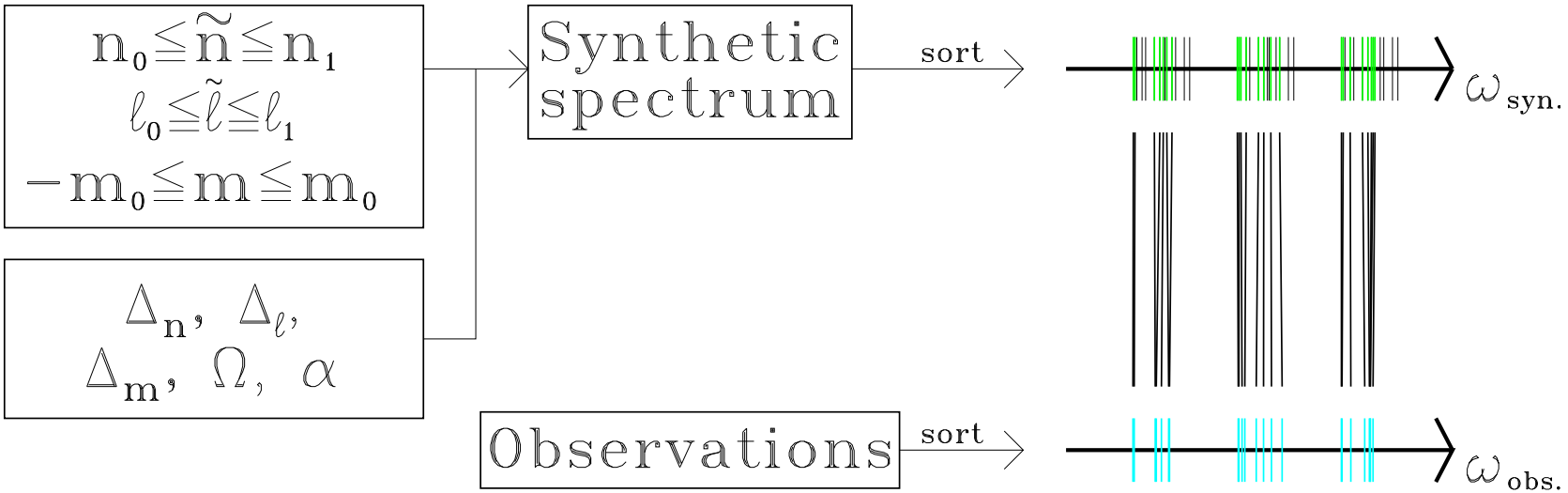

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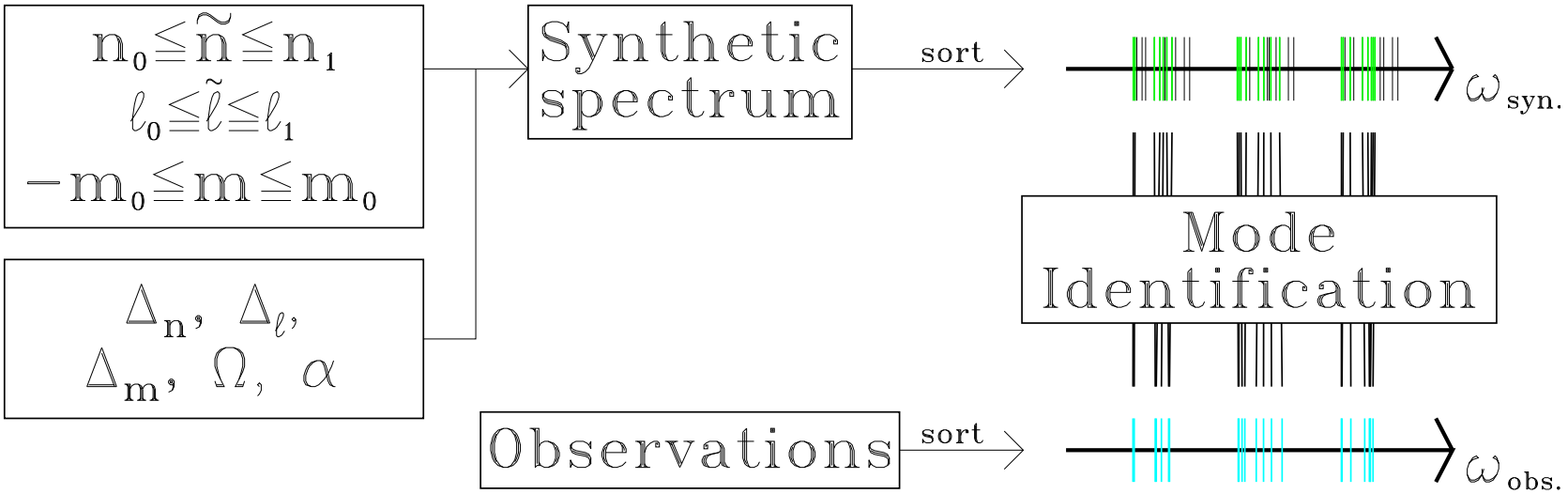

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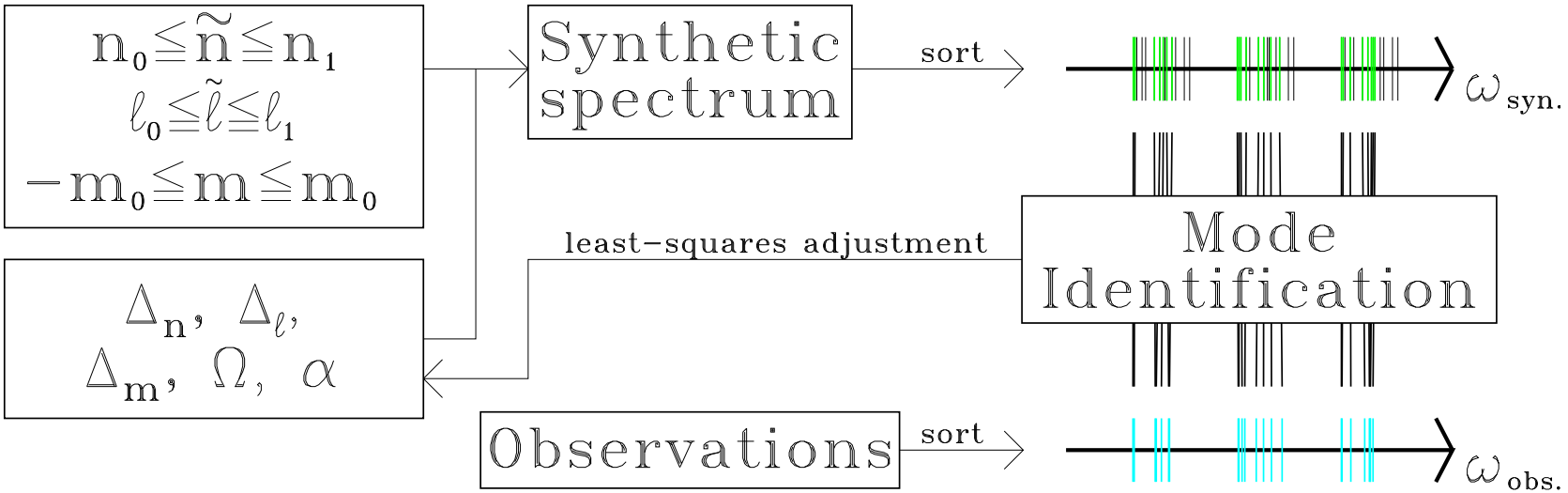

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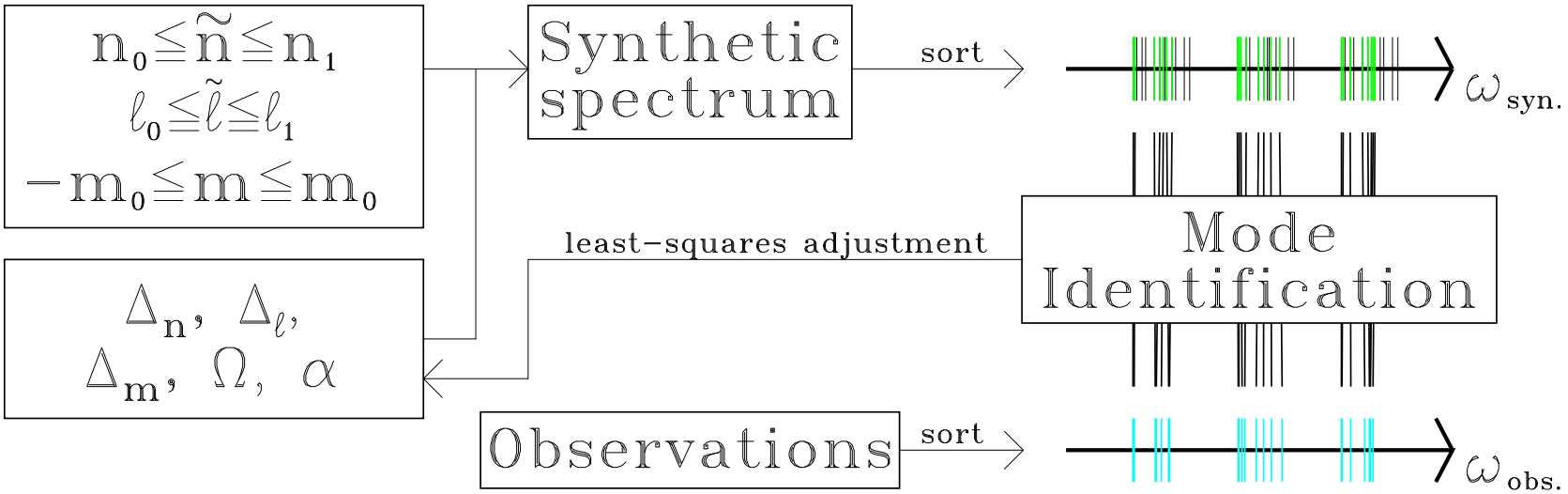

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- this yields a mode identification
- this can then be used to readjust $\Delta_{\tilde{n}}$, $\Delta_{\tilde{\ell}}$, $\Delta_{\tilde{m}}$, α, Ω
- adjust parameters and obtain a closer fit to observed frequencies

General procedure

- the previous slide explained how to find a plausible mode identification for a given set of parameters
- repeat the same procedure while scanning a 5D parameter space
- a slightly different set of parameters is used:

$$\Delta_{\tilde{n}}, \quad \frac{\Delta_{\tilde{\ell}}}{\Delta_{\tilde{n}}}, \quad \frac{\Delta_{\tilde{m}}}{\Delta_{\tilde{n}}}, \quad \frac{\alpha}{\Delta_{\tilde{n}}}, \quad \frac{\Omega}{\Delta_{\tilde{n}}}$$

  - the synthetic spectrum does not need to be sorted again when $\Delta_{\tilde{n}}$ or $\frac{\alpha}{\Delta_{\tilde{n}}}$ change value

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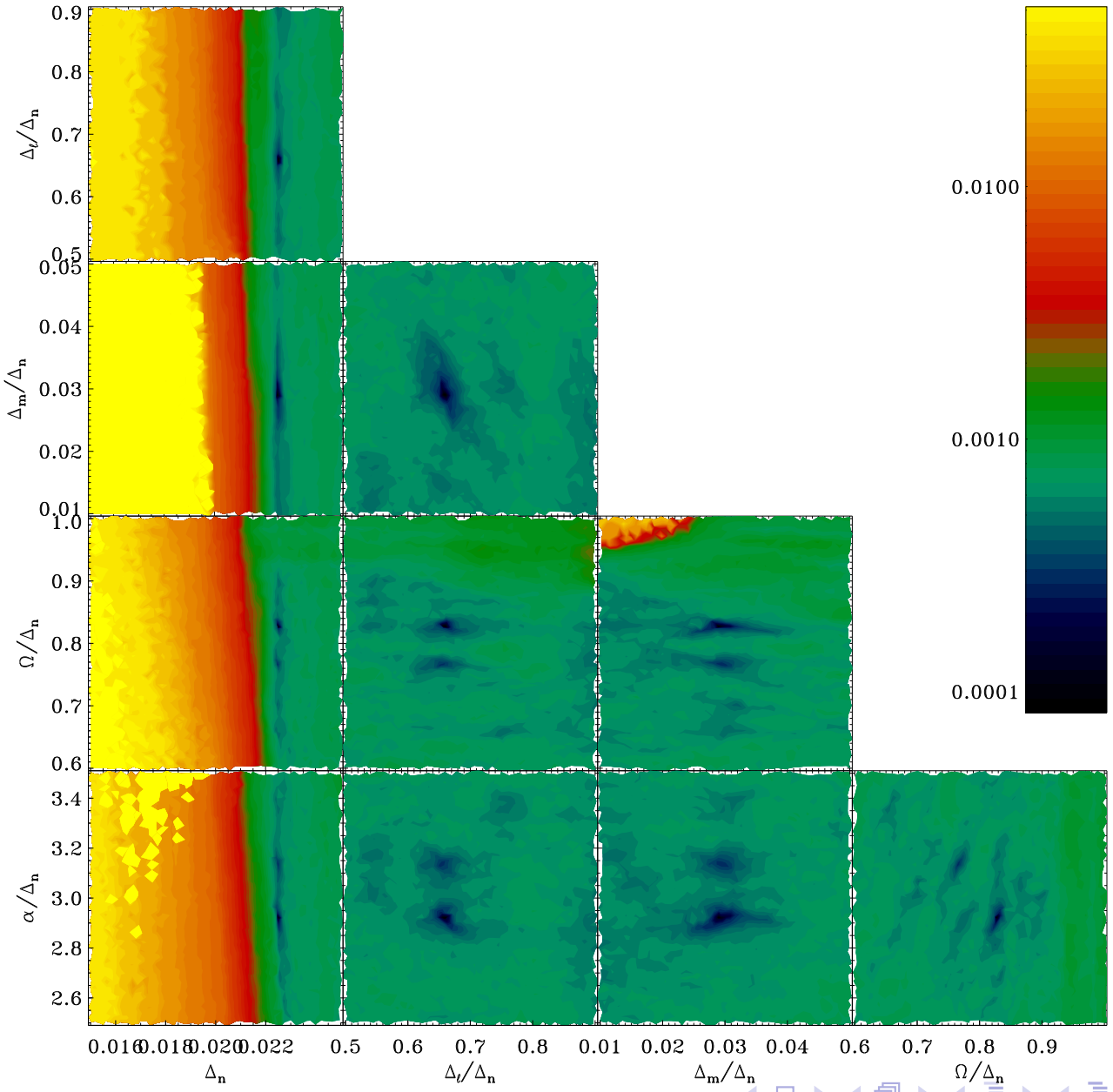
Computational cost

- about 1 hour for 50^5 parameter combinations

Synthetic "Observations"



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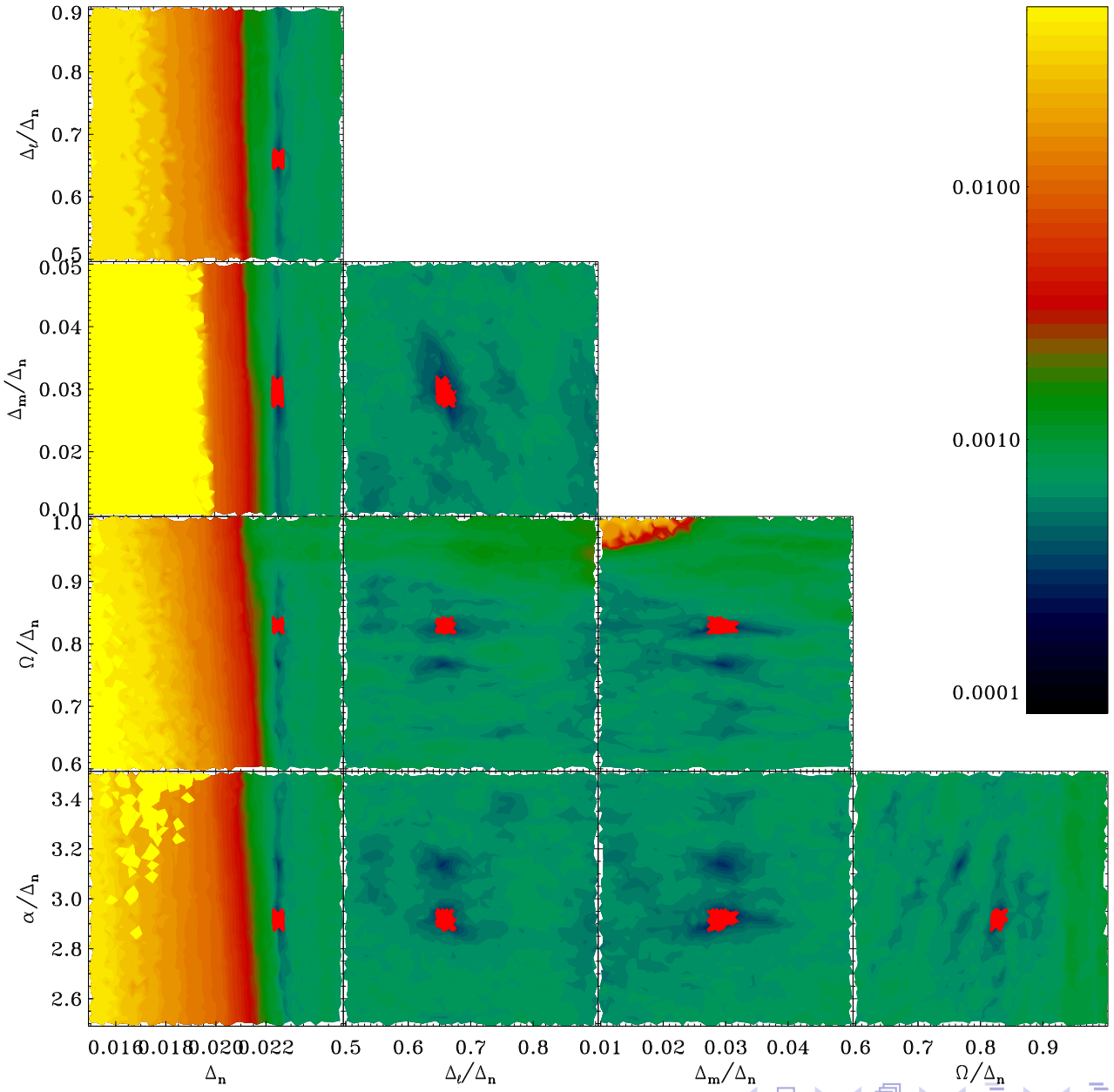

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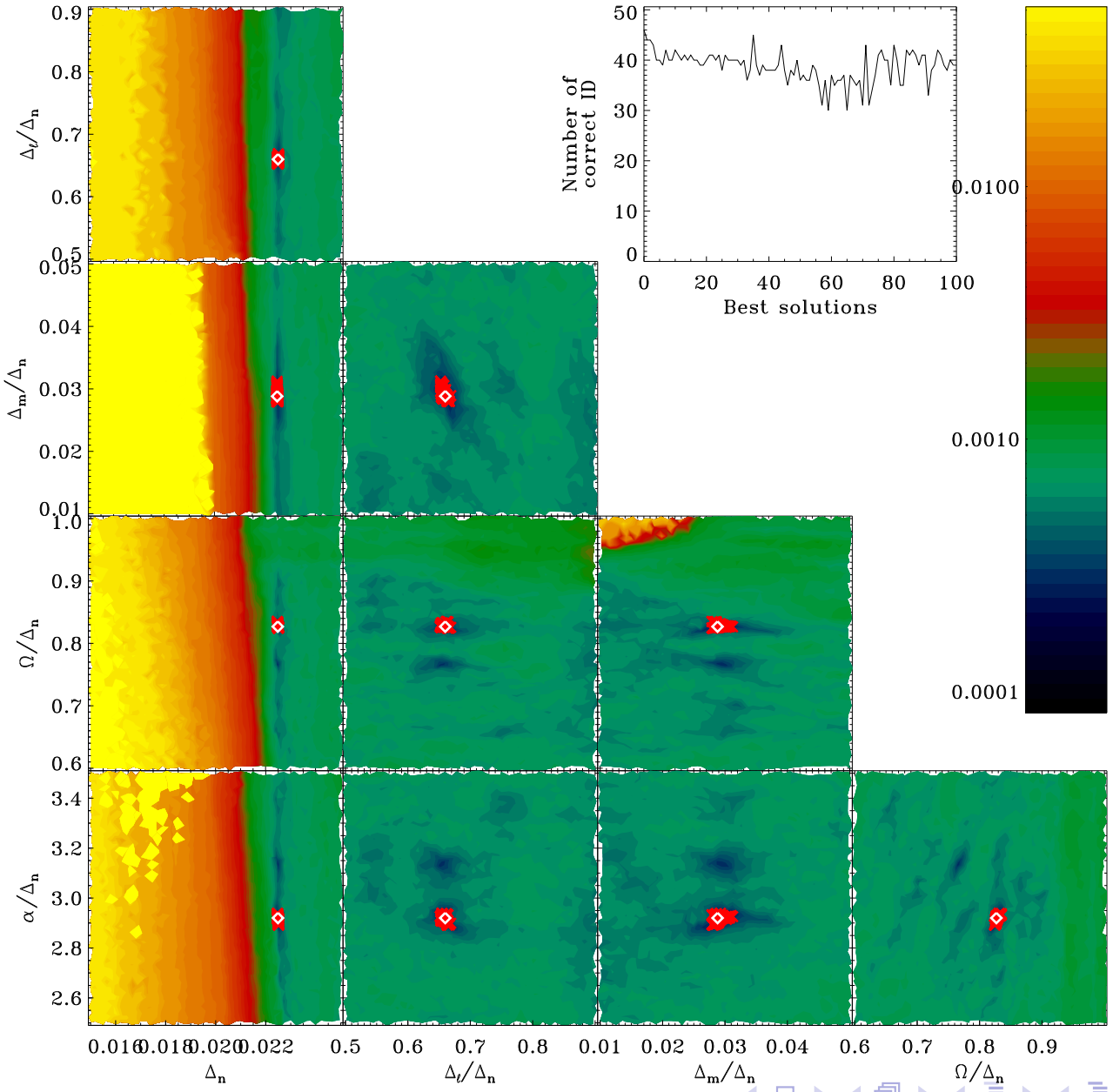
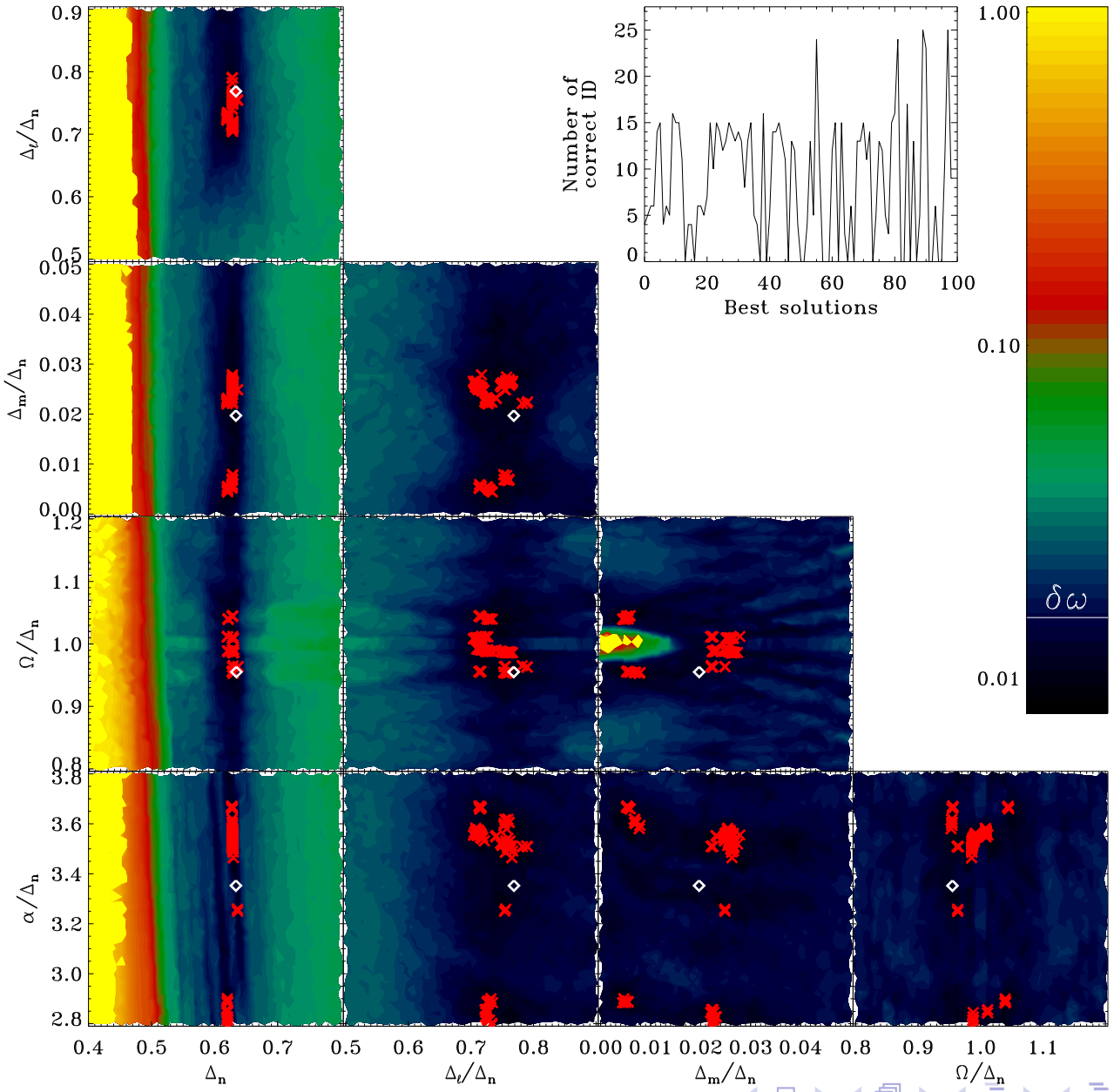
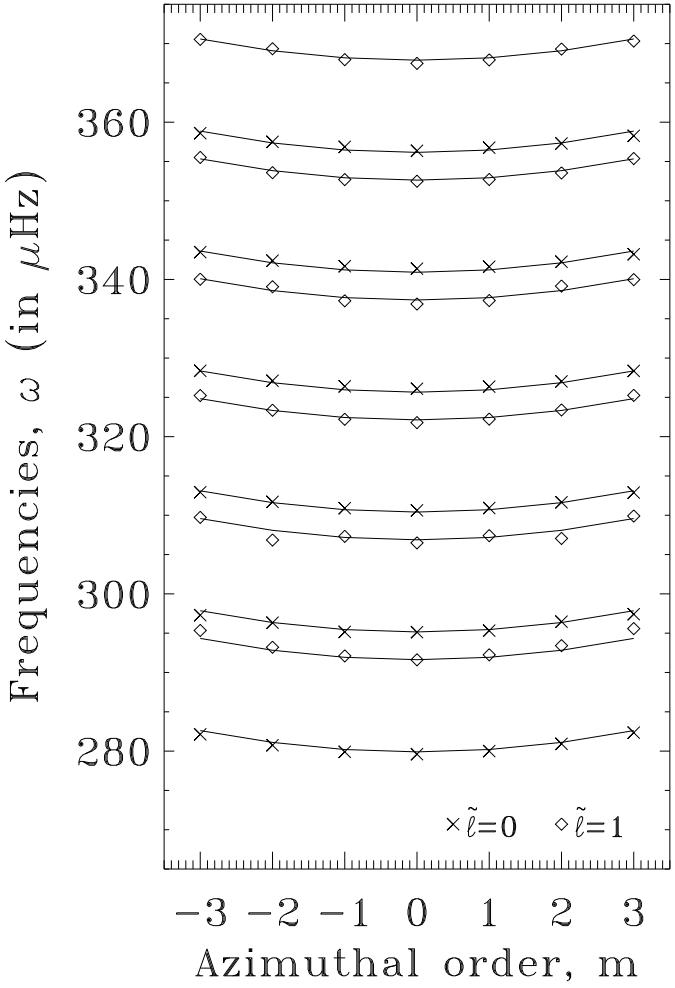
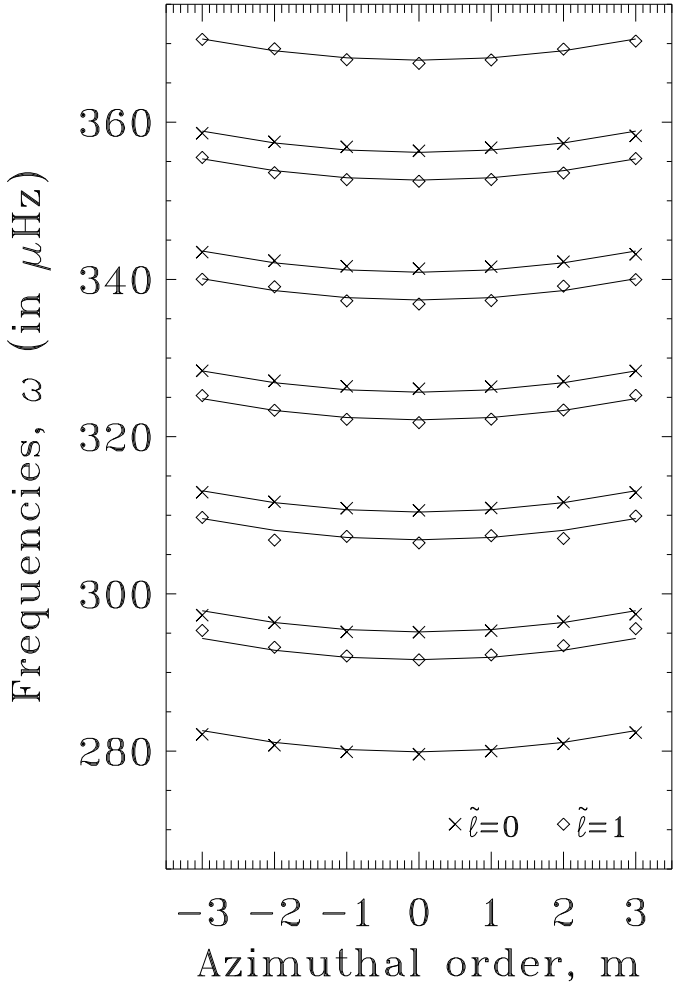


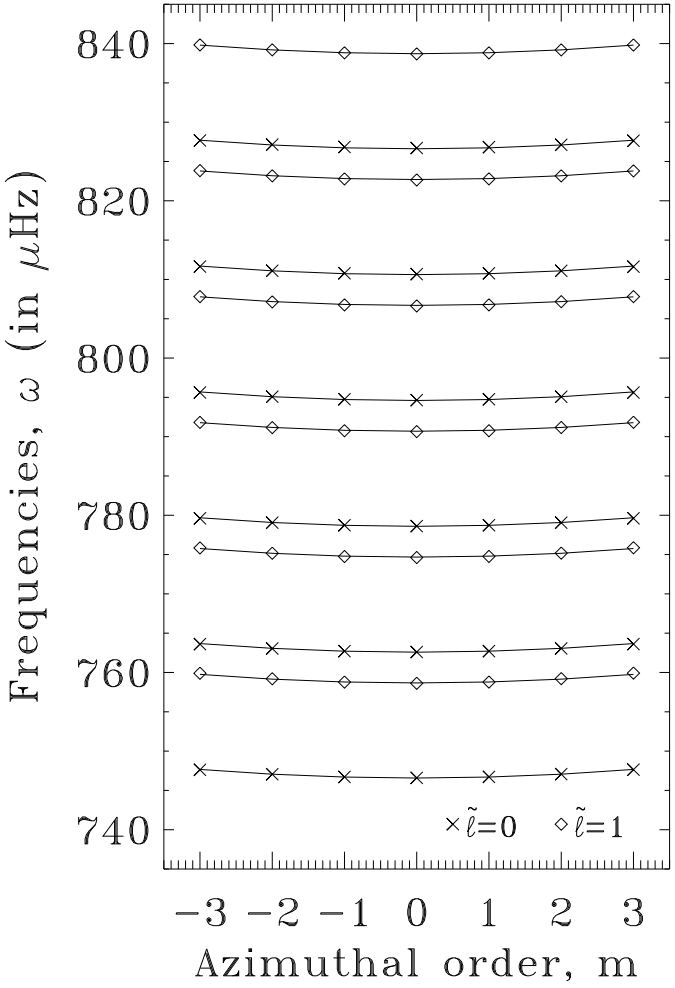
Numerical frequencies





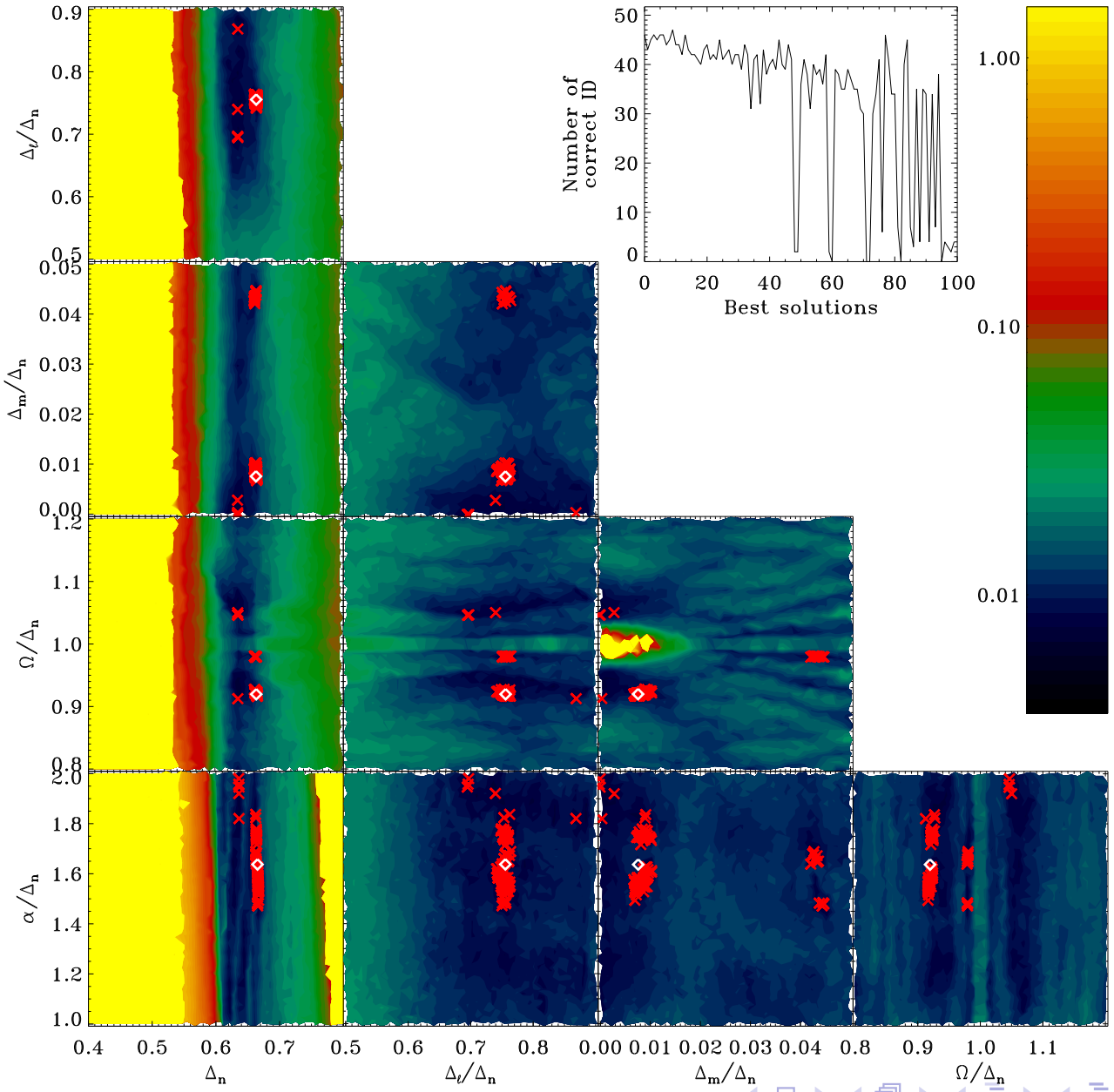
- avoided crossings cause deviations between asymptotic formula and numerical frequencies





- avoided crossings cause deviations between asymptotic formula and numerical frequencies
- these deviations become much smaller at high radial orders

Numerical frequencies



- the previous example all contained 50 observed modes
- what happens when you change the number of modes?

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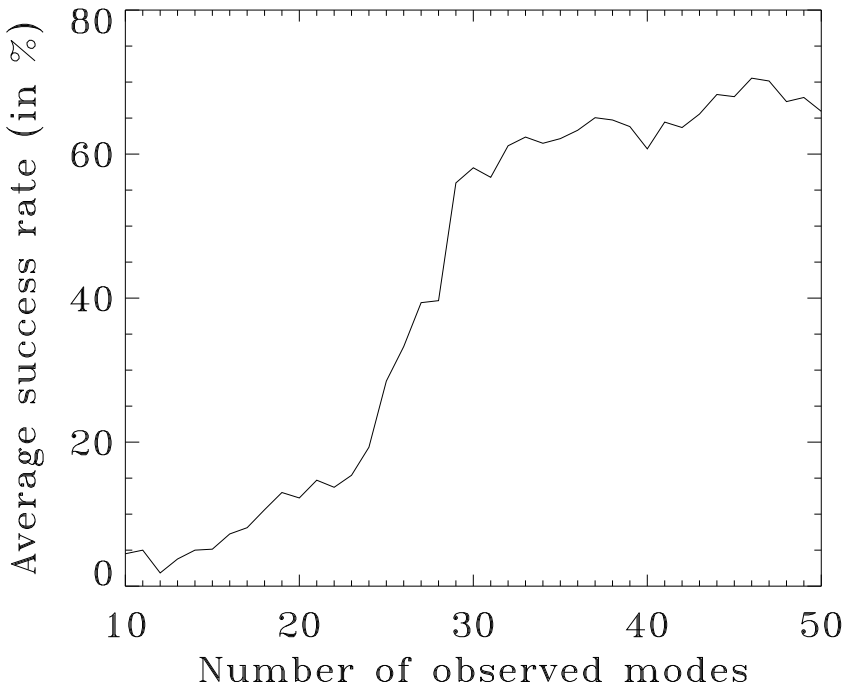

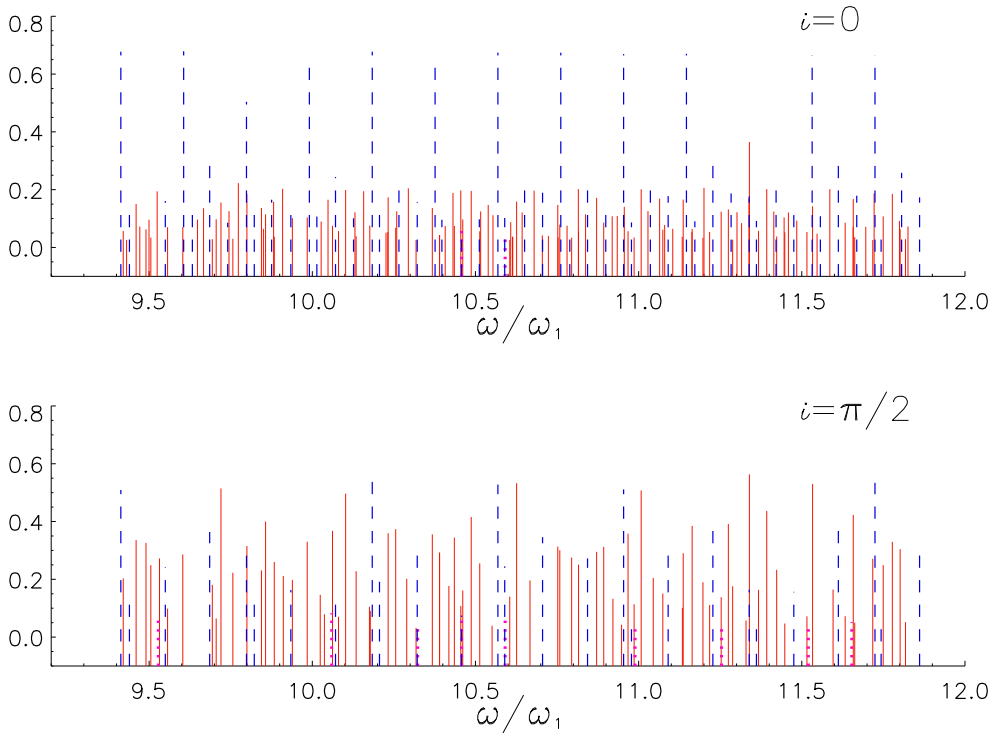

Lignières & Georgeot, 2009, submitted to A&A

- chaotic modes are also likely to be visible in the frequency spectrum

- what happens to mode identification?

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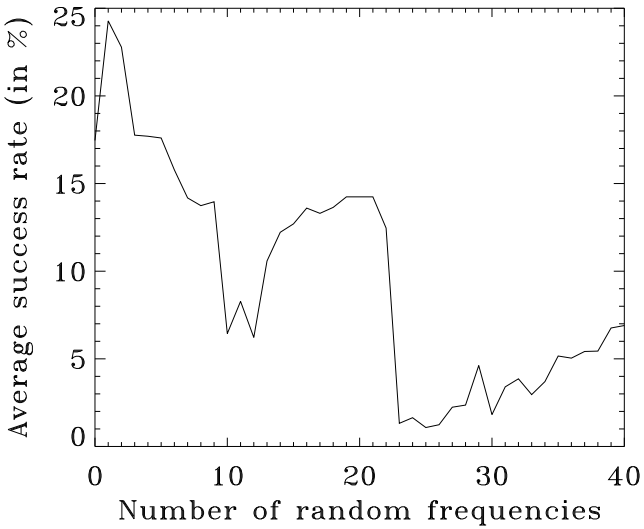
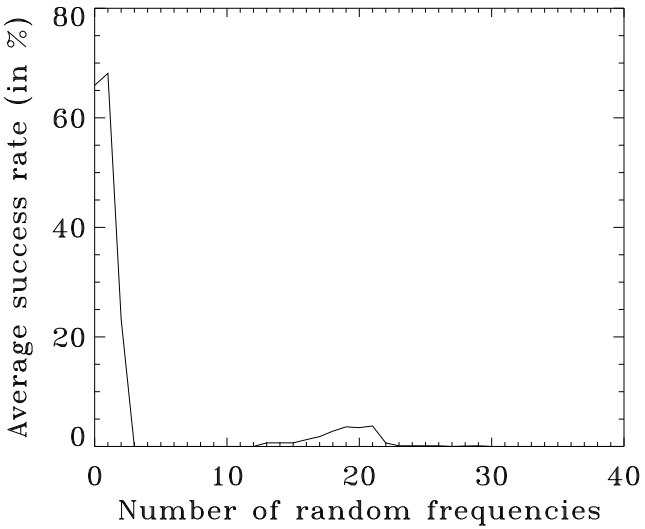
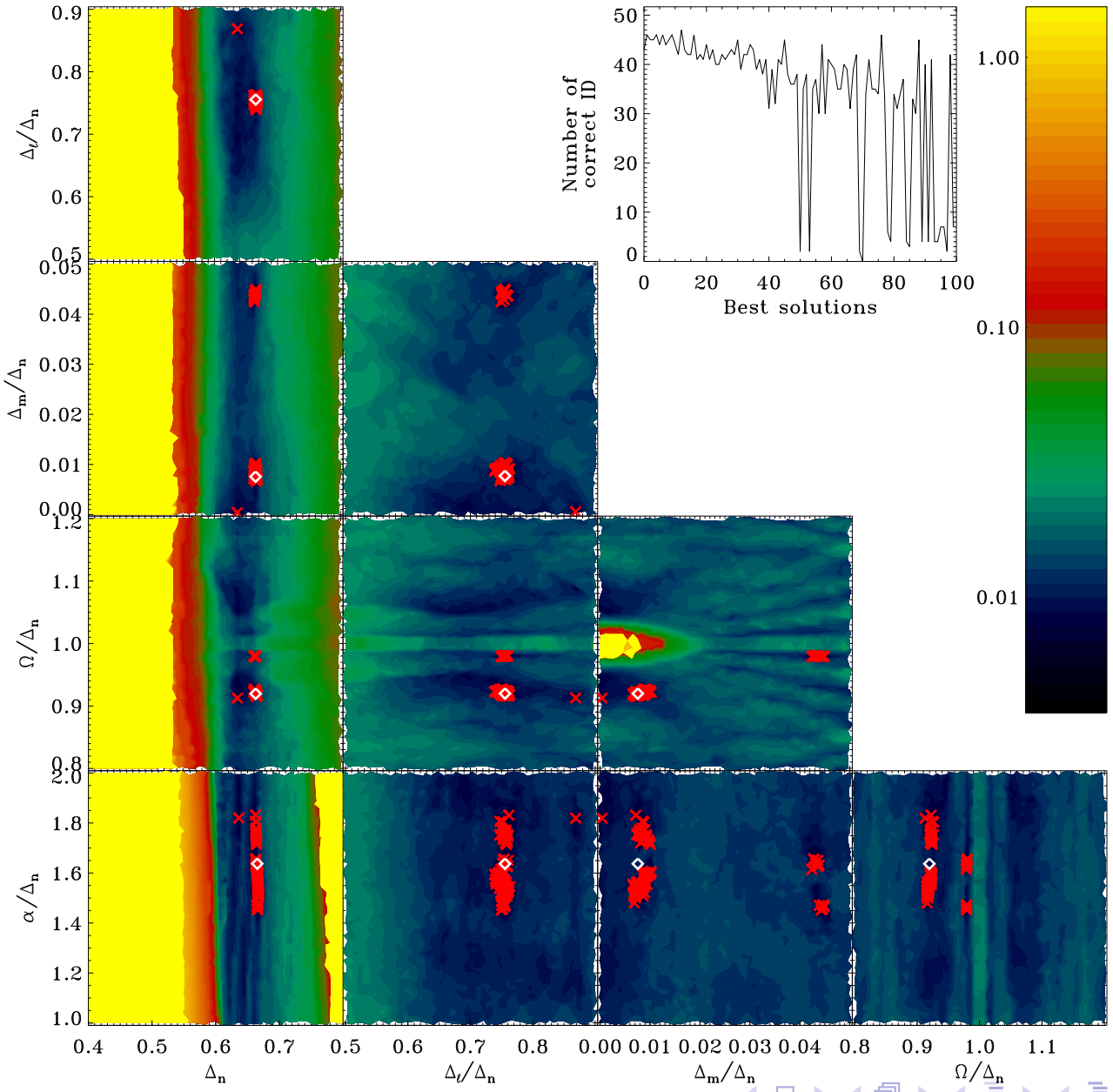


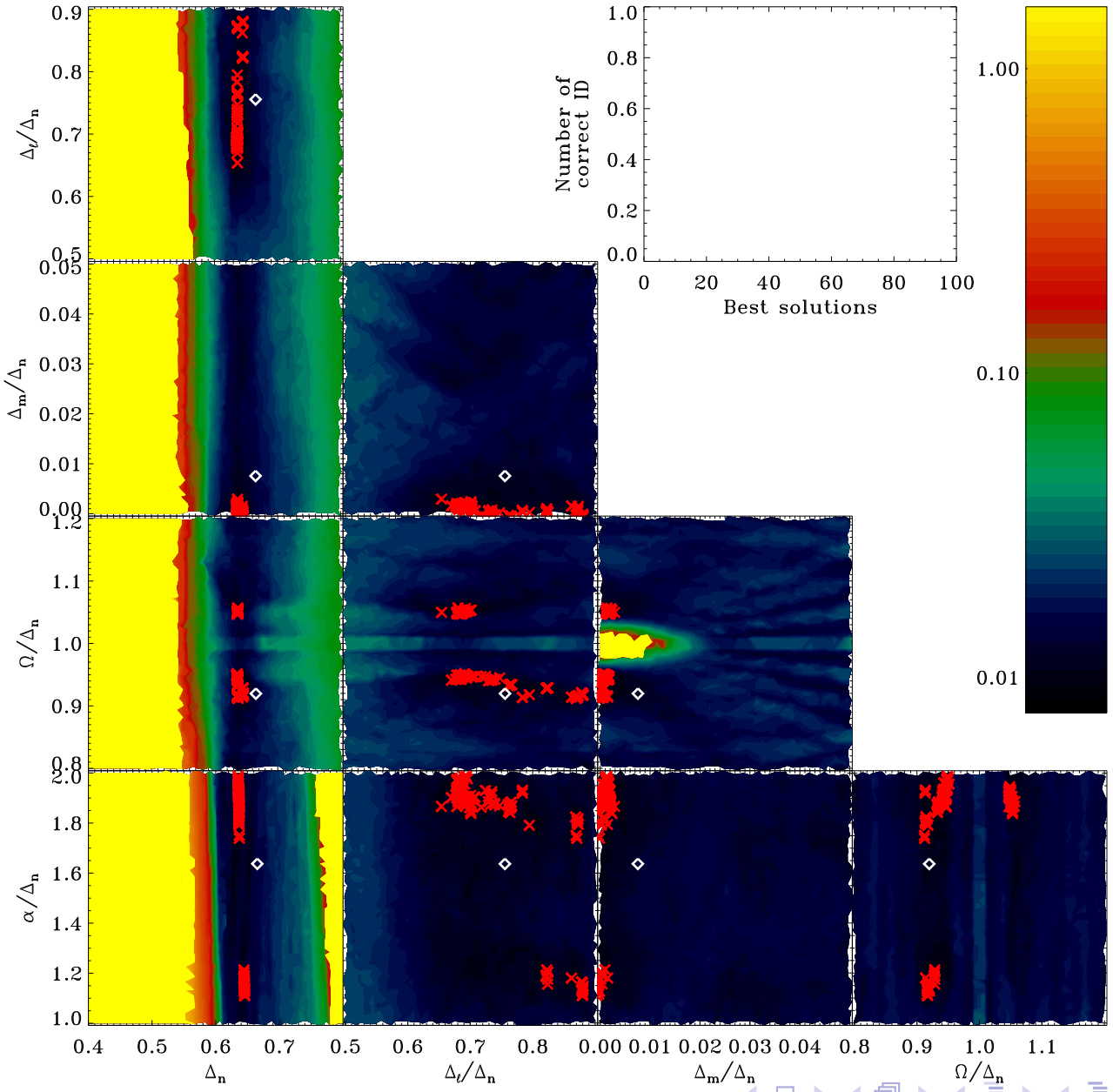


- low radial orders
- high radial orders

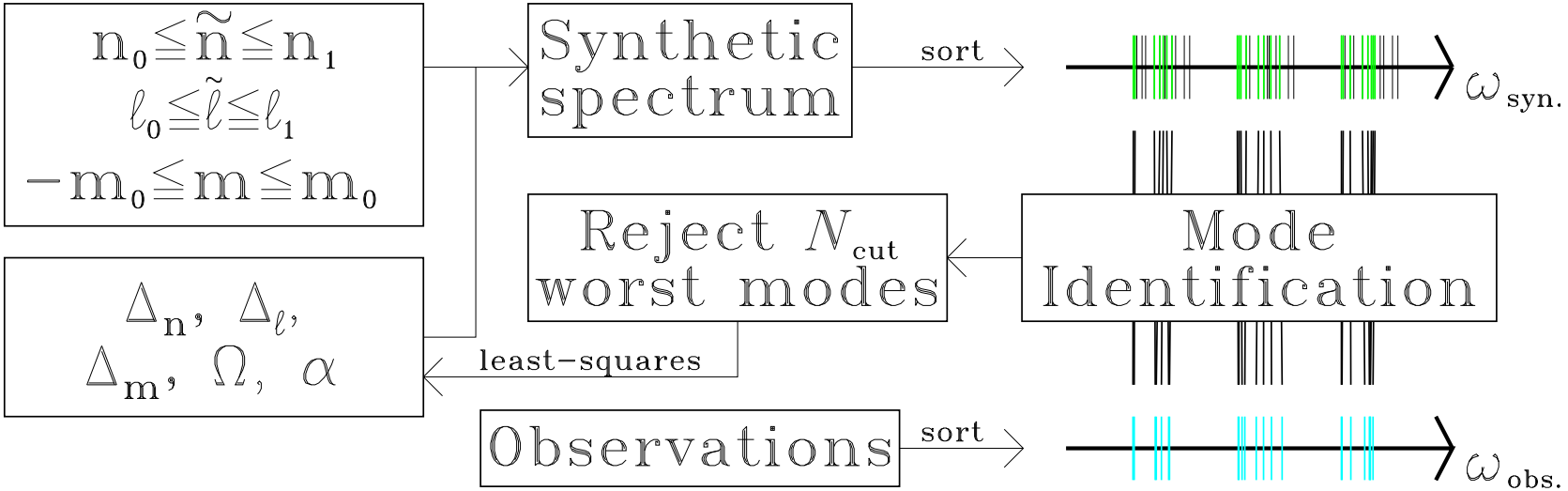
1 random frequency



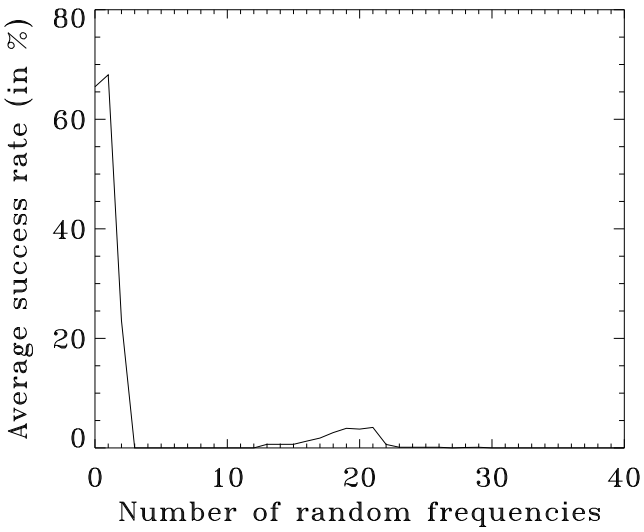
3 random frequencies

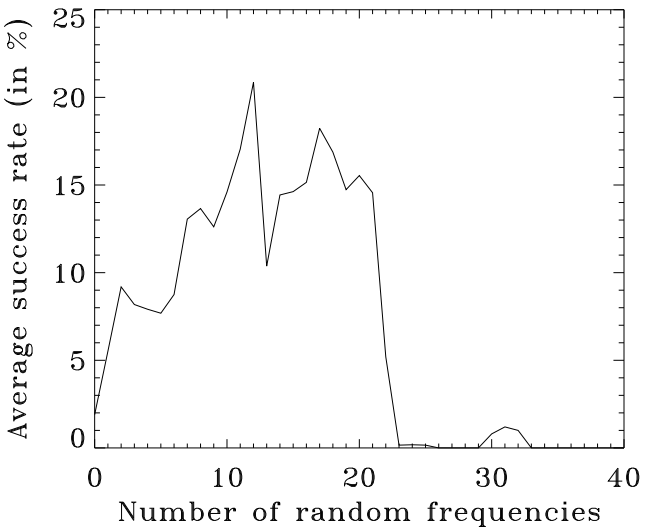


Improving the mode identification scheme



- allow the algorithm to reject the N_{cut} worst solutions



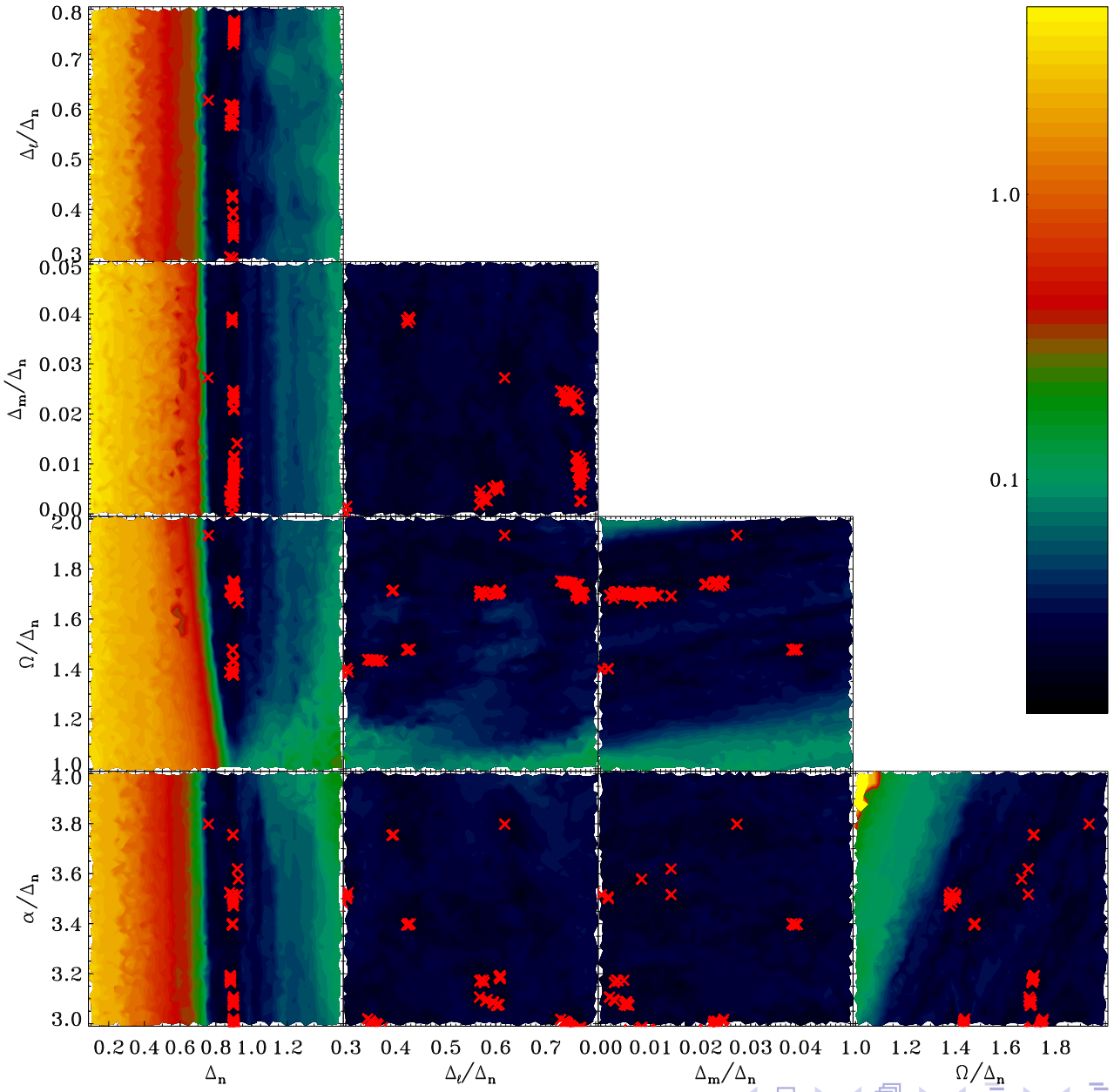


- keeping all frequencies
- rejecting 20 frequencies

Conclusion

- it is possible to identify acoustic modes in rapidly rotating stars based on the asymptotic formula for island modes
  - a sufficient number of modes is needed
  - modes with high radial orders yield better results
  - chaotic modes are problematic
- possible improvements:
  - correlate theoretical mode visibilities with observed mode amplitudes
  - use results from 2D pulsation calculations to restrict parameter space
  - take non-adiabatic effects into account to know which modes are excited
- direct comparison between observations and numerical calculations

ζ Ophiuchi: 19 frequencies



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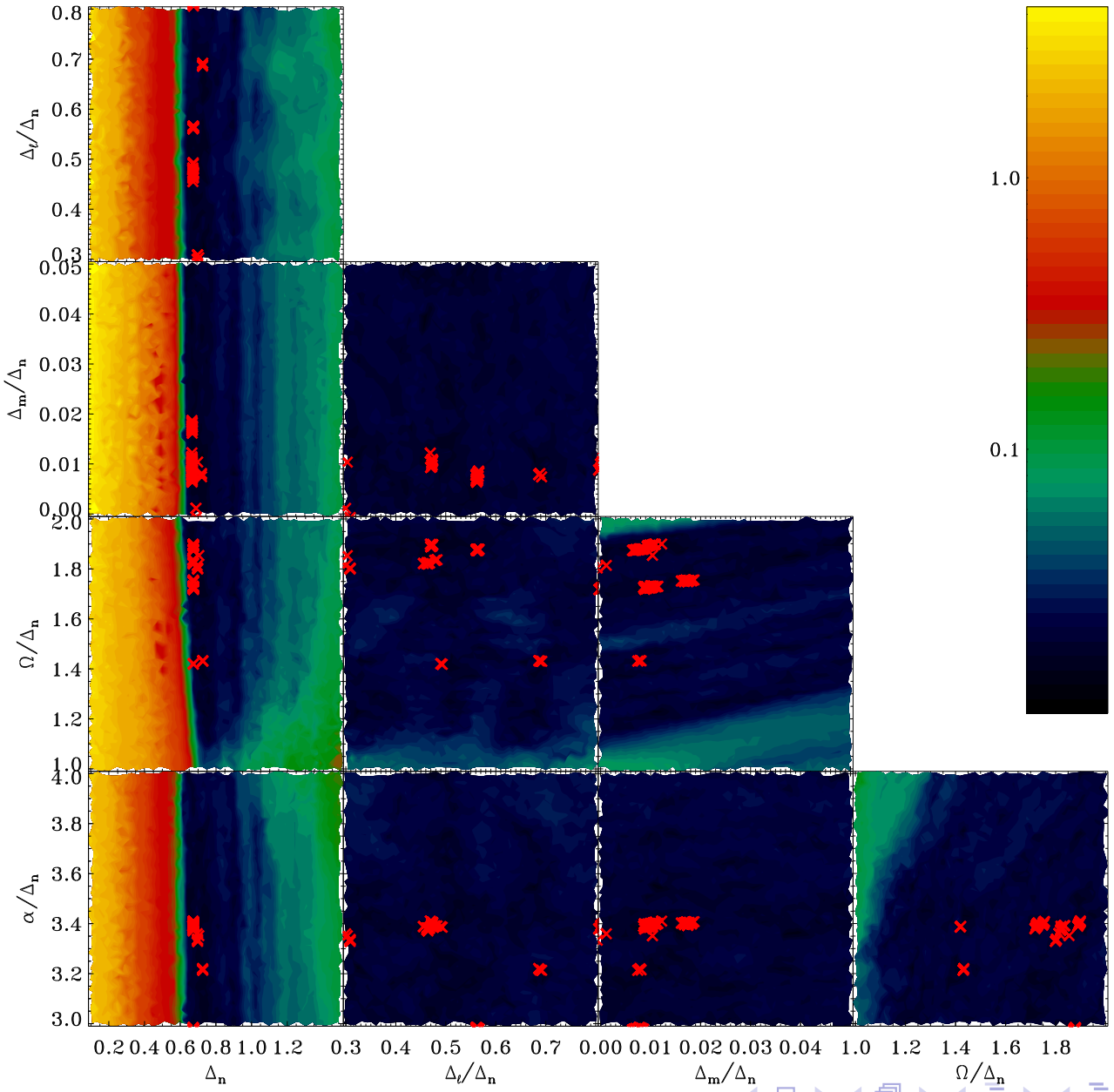
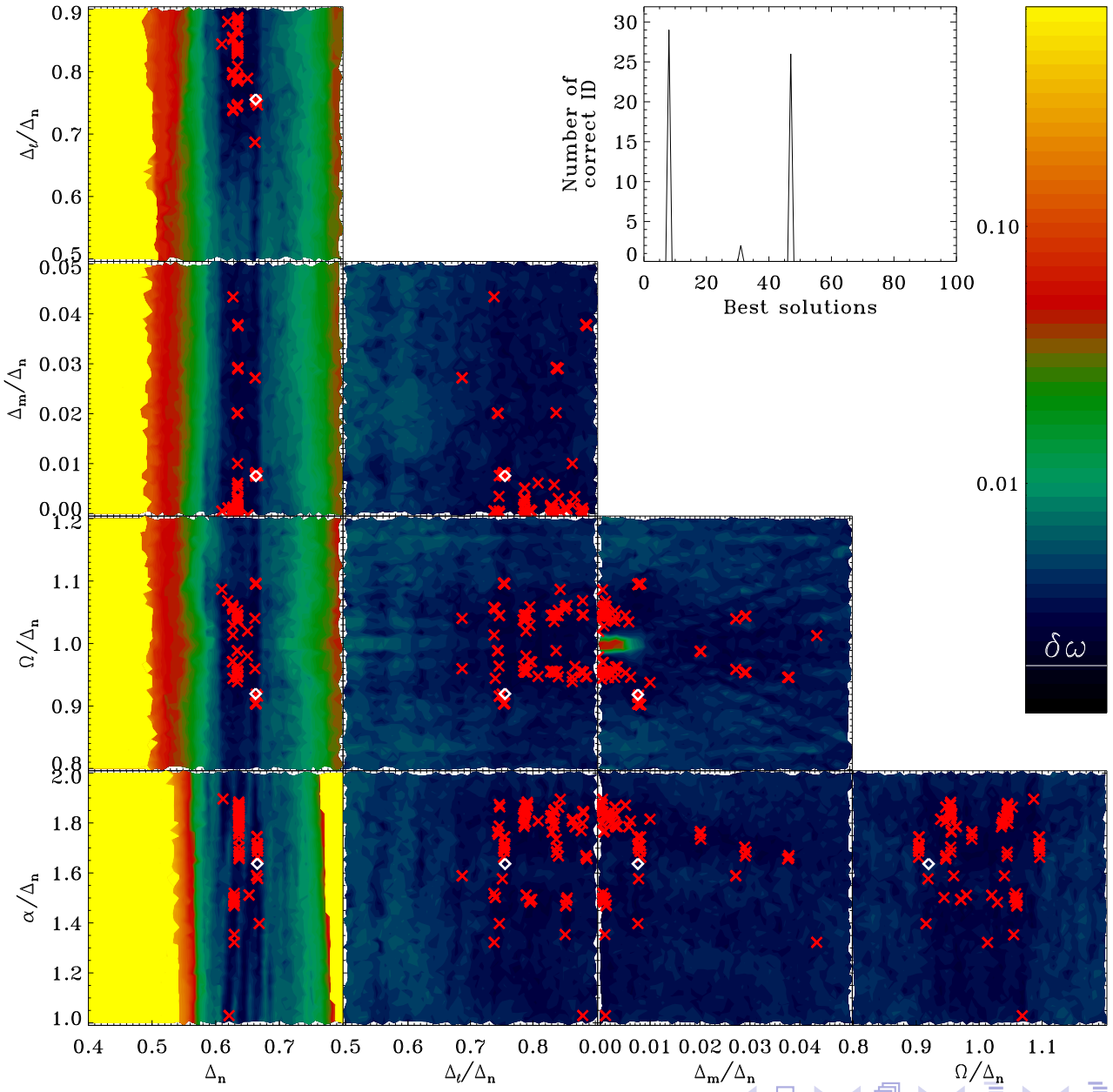


$N_{cut} = 20,\ N_{rand} = 0$



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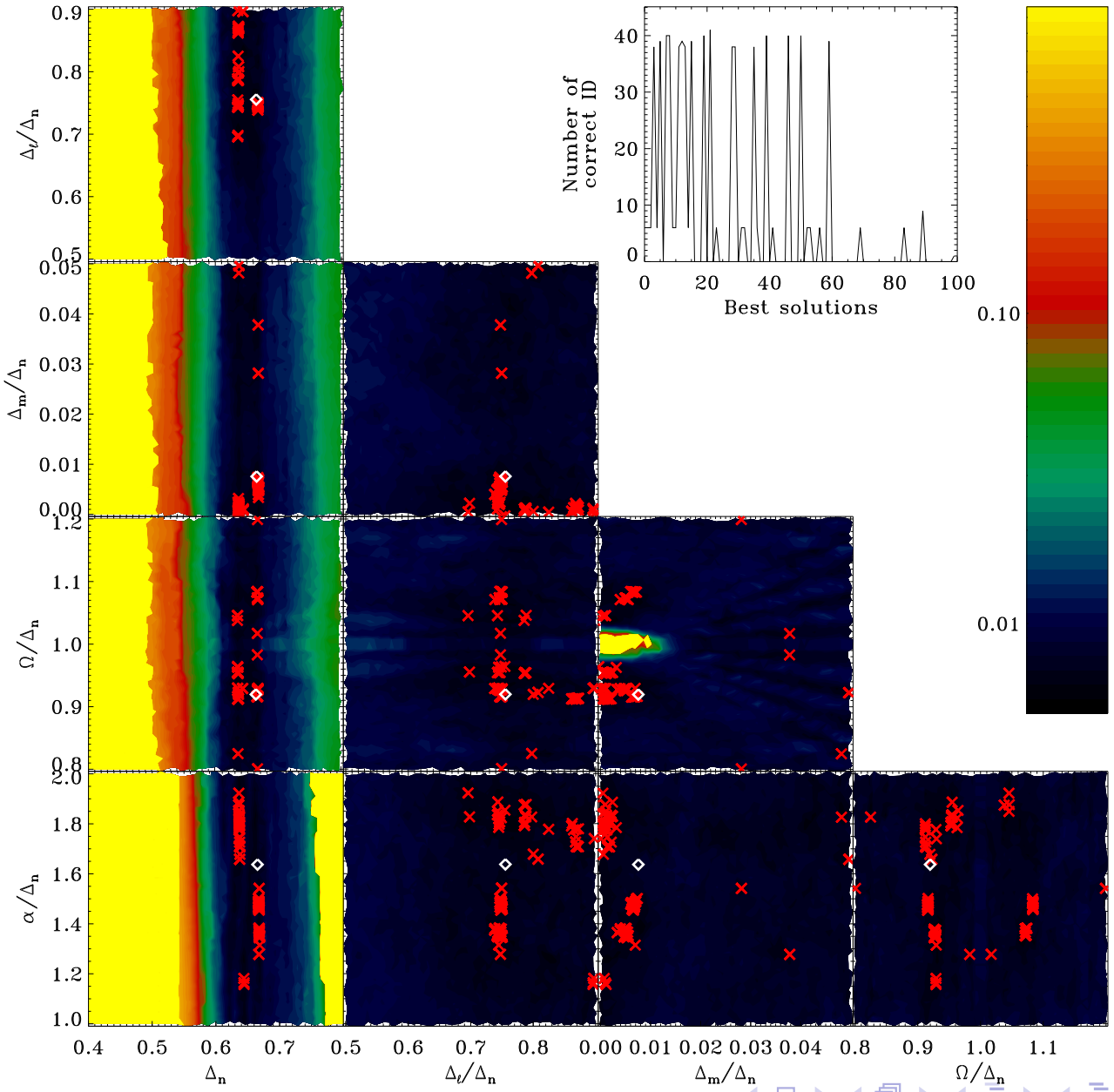
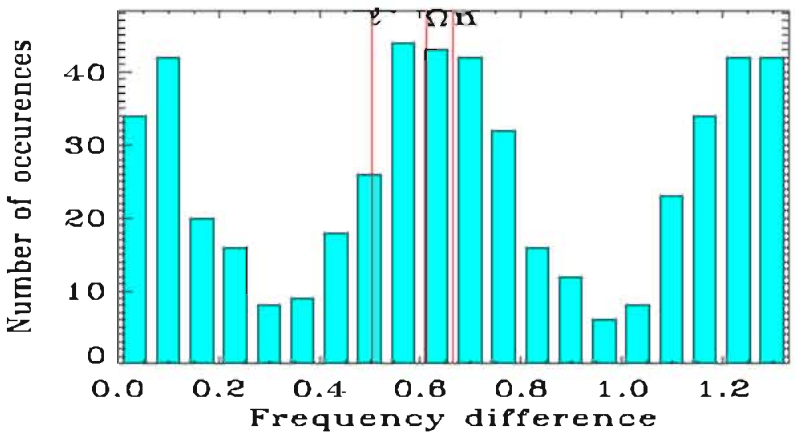
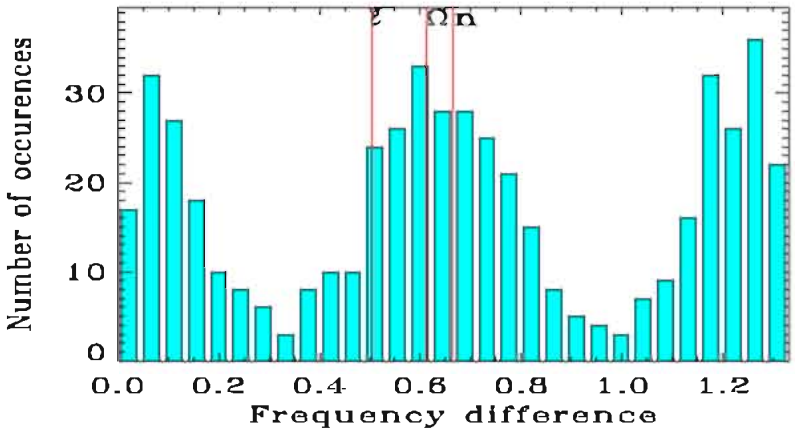



Number of occurences
Frequency difference
0.0 0.2 0.4 0.6 0.8 1.0 1.2
0 10 20 30 40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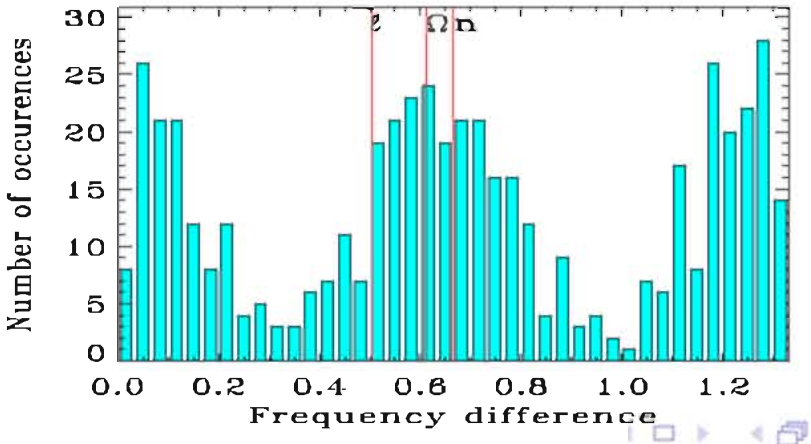
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