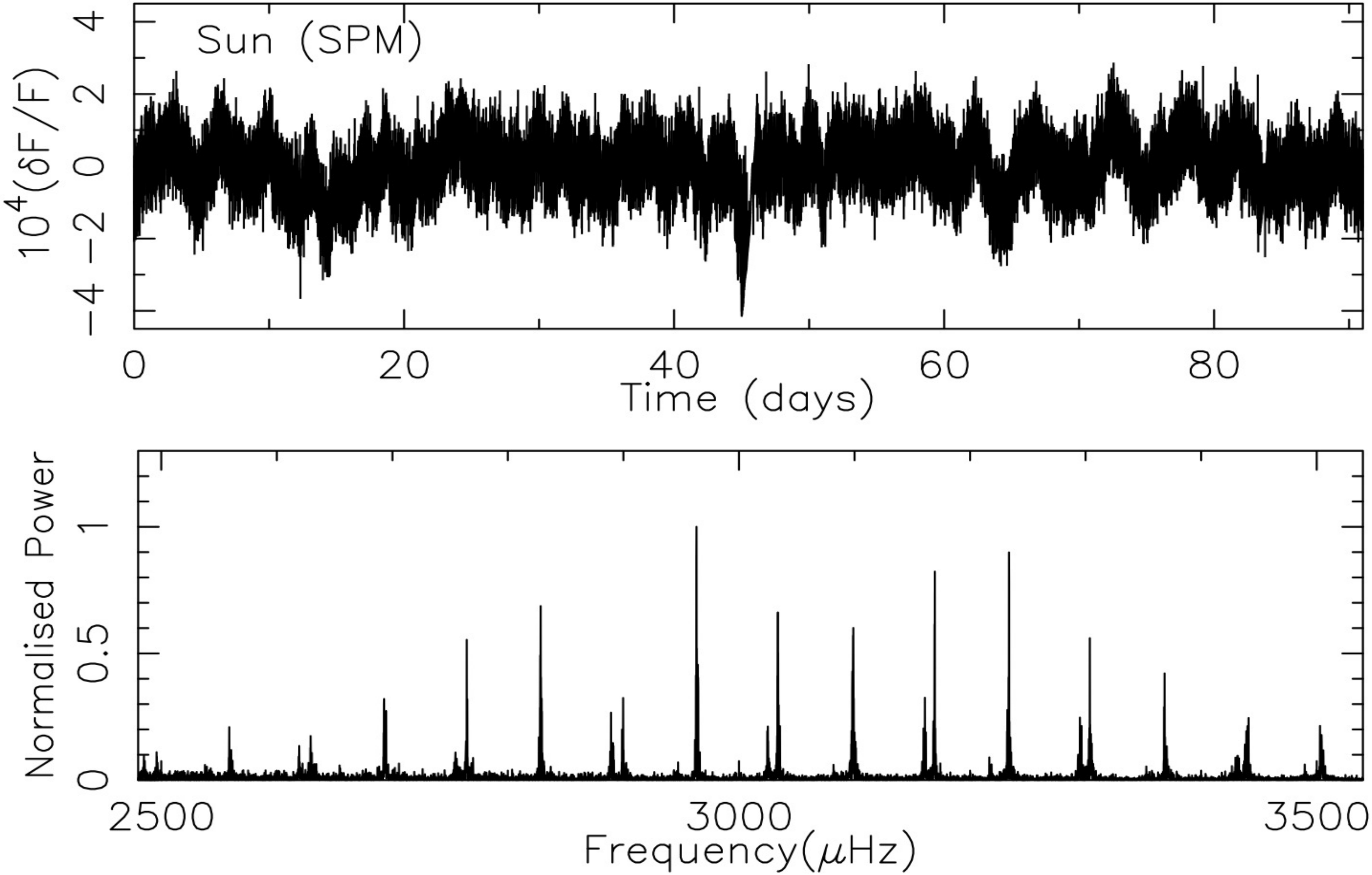
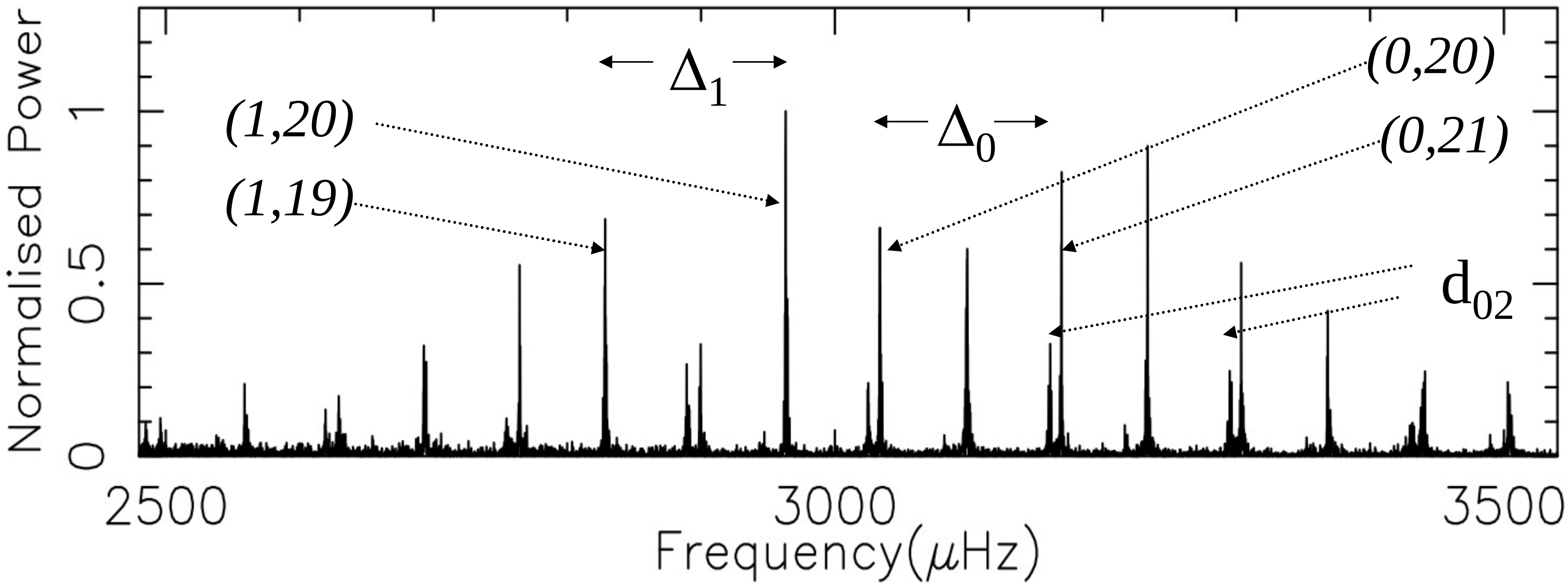
Separations and phase shift differences of $l = 0,1$ p-modes

Solar Time series and Power spectrum



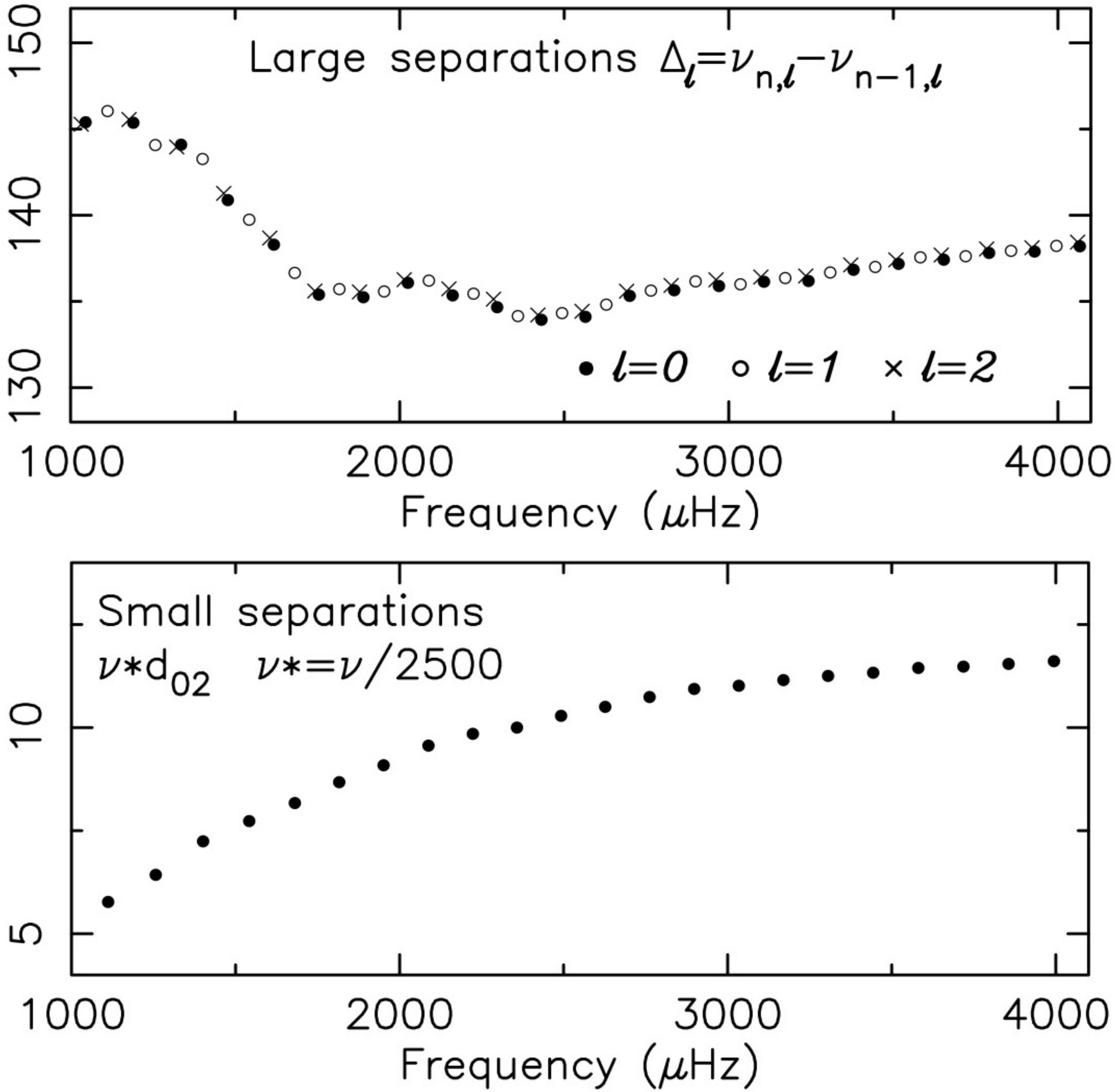
Large Δ and small d separations



Small separations: $\Delta_0(n) = \nu_{n,0} - \nu_{n\text{-}1,0}$ $\Delta_1(n) = \nu_{n,1} - \nu_{n\text{-}1,1,}$ *etc*

Small separations: $d_{02}(n) = \nu_{n,0} - \nu_{n\text{-}1,2}$ $d_{13}(n) = \nu_{n,1} - \nu_{n\text{-}1,3}$

Classical separations



$\Delta_0(n) = \nu_{n,0} - \nu_{n-1,0}$

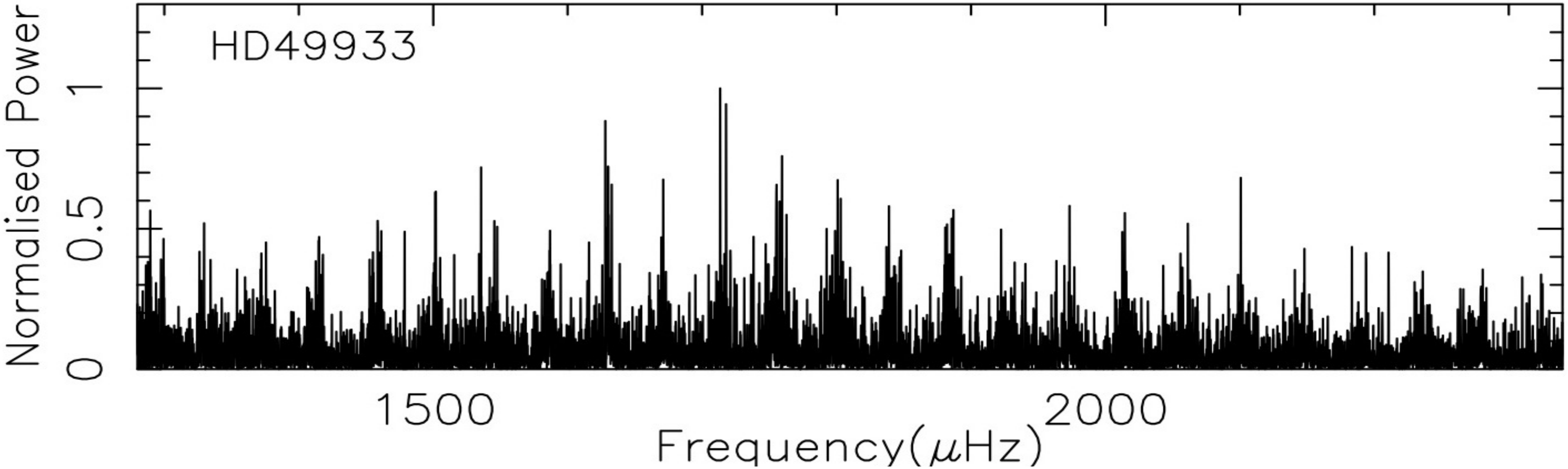
$\Delta_1(n) = \nu_{n,1} - \nu_{n-1,1}$, *etc*

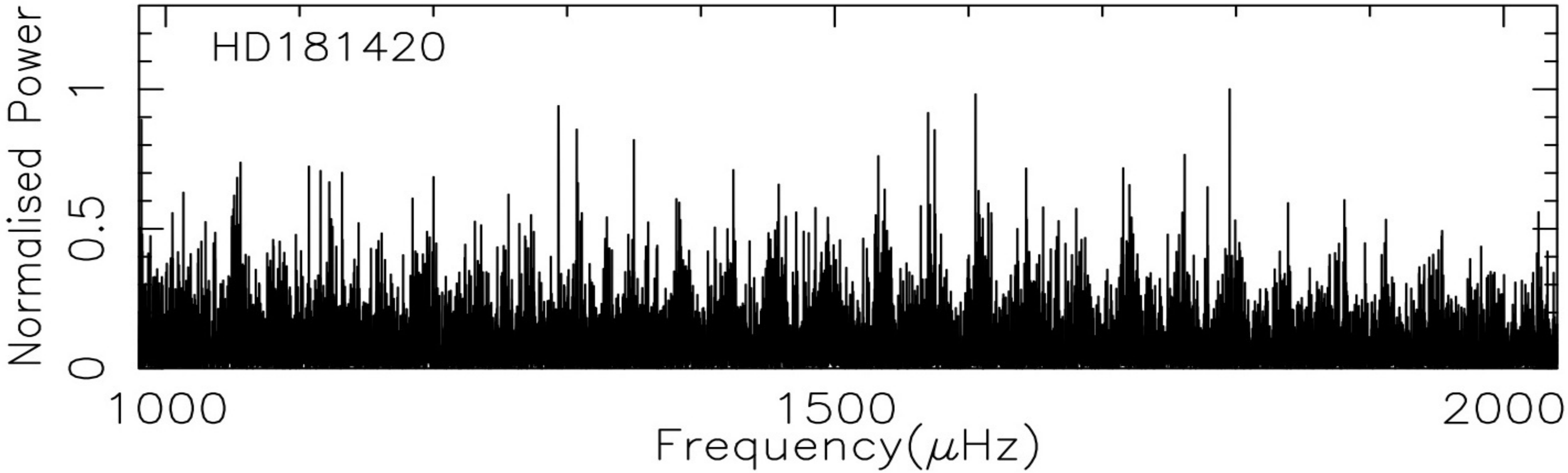
$d_{02}(n) = \nu_{n,0} - \nu_{n-1,2}$

$d_{13}(n) = \nu_{n,1} - \nu_{n-1,3}$

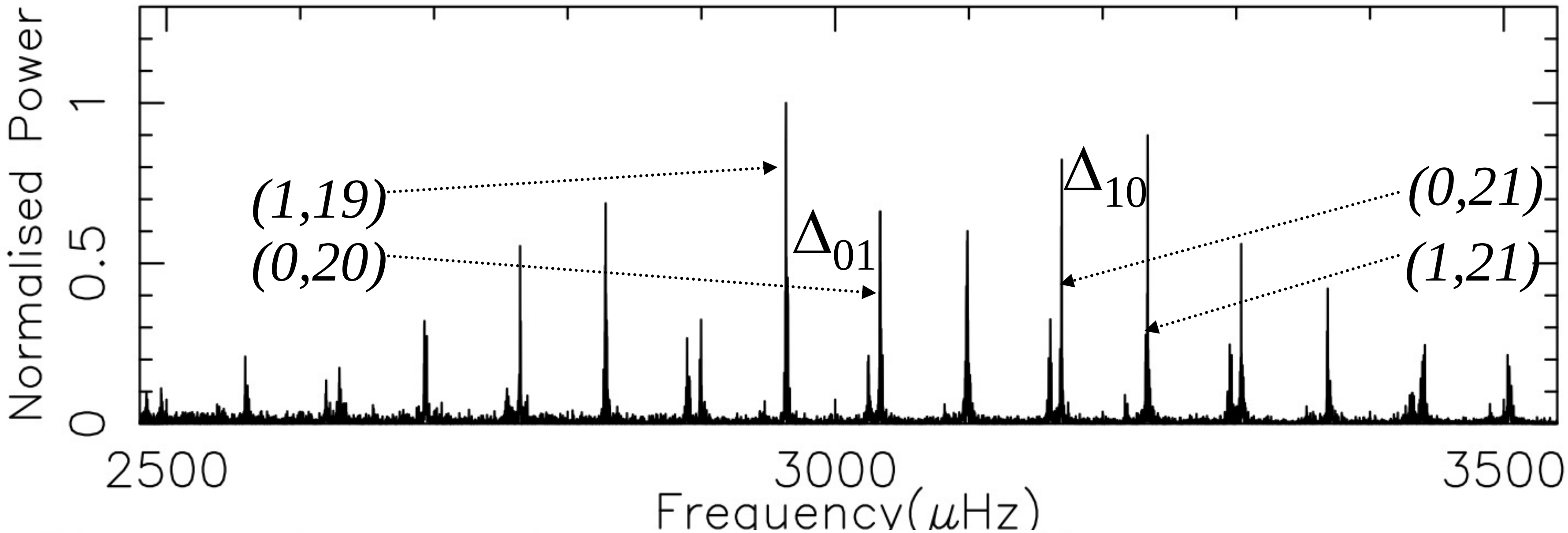
Solar model A

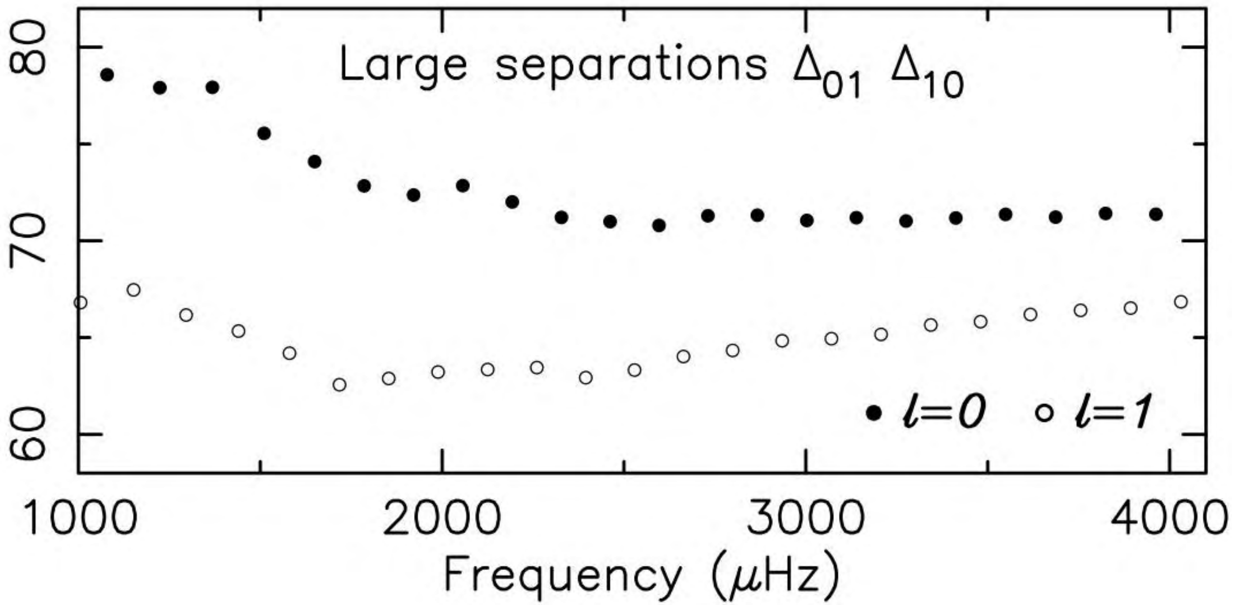
CoRoT stars are not easy - probably only l=0,1 modes





l=0,1 "Large" separations





$$\Delta_{01}(n) = \nu_{n,0} - \nu_{n-1,1}$$

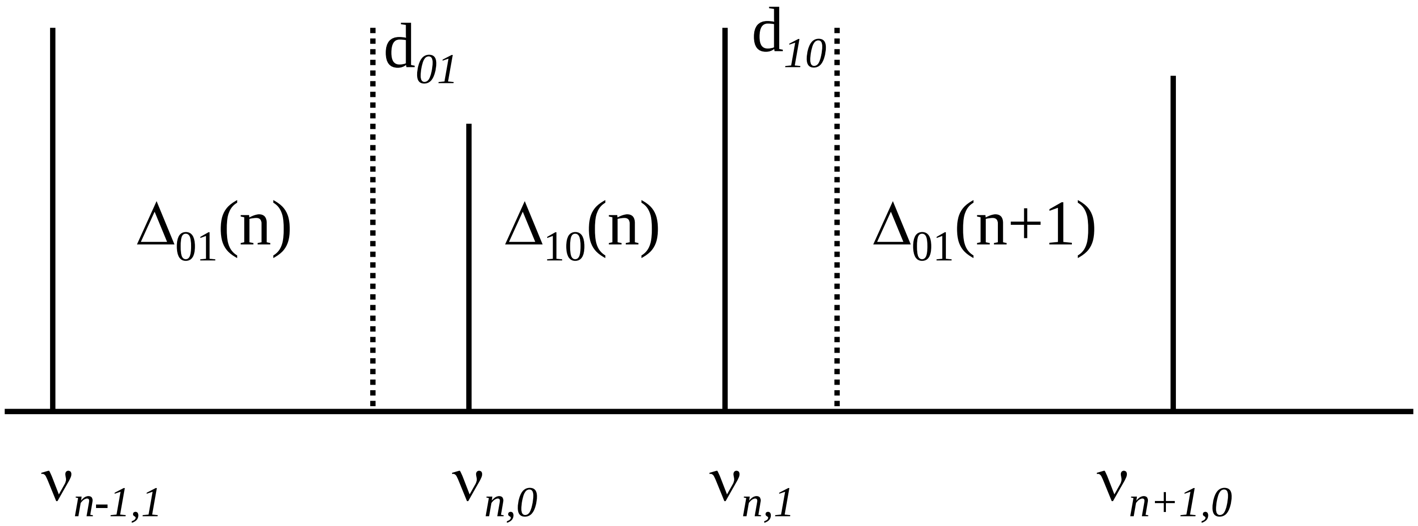
$$\Delta_{10}(n) = \nu_{n,1} - \nu_{n,0}$$

$$\Delta_{01} \neq \Delta_{10}$$

systematic and modulated?

Solar model A

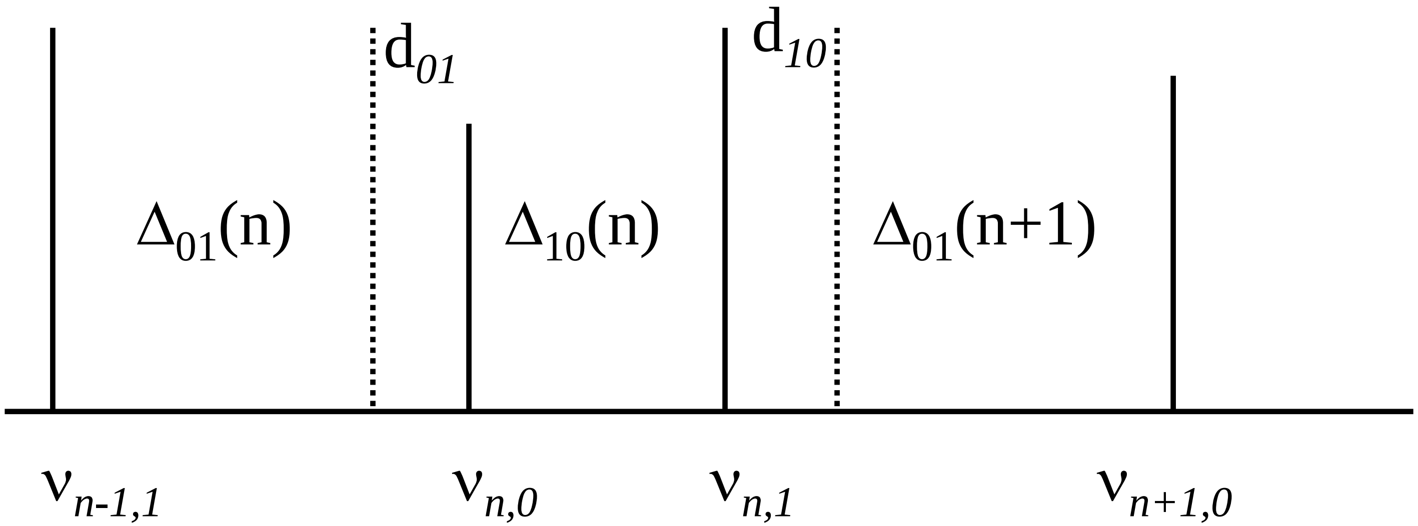
l=0,1 Small Separations



$$\Delta_{01}(n) - \Delta_{10}(n) \quad \Rightarrow \quad d_{01}(n) = \nu_{n,0} - (\nu_{n-1,1} + \nu_{n,1})/2$$

$$\Delta_{01}(n+1) - \Delta_{10}(n) \quad \Rightarrow \quad d_{10}(n) = (\nu_{n,0} + \nu_{n+1,0})/2 - \nu_{n,1}$$

l=0,1 Small Separations

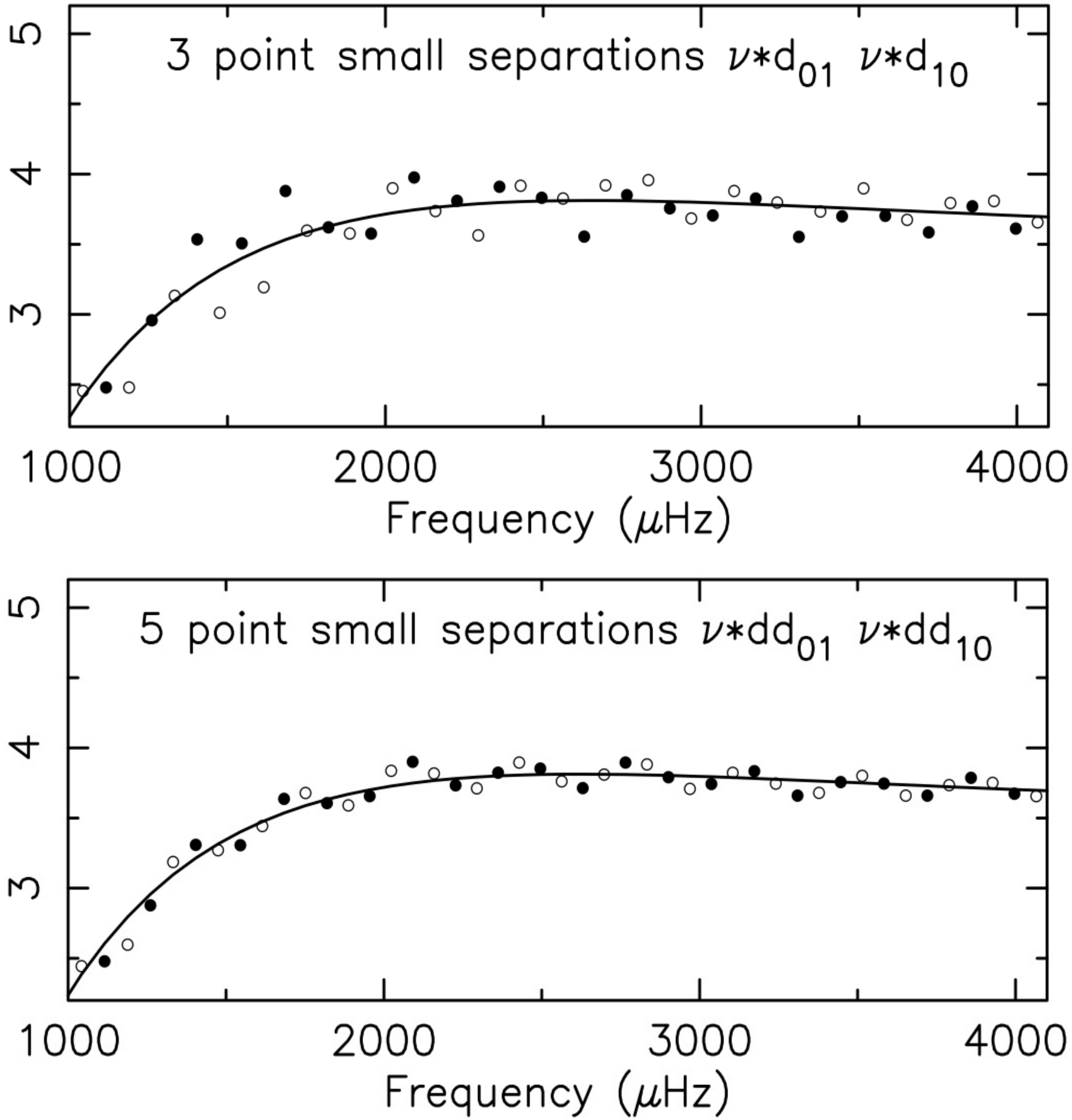


$$\Delta_{01}(n) - \Delta_{10}(n) \quad \Rightarrow \quad d_{01}(n) = \nu_{n,0} - (\nu_{n-1,1} + \nu_{n,1})/2$$

$$\Delta_{01}(n+1) - \Delta_{10}(n) \quad \Rightarrow \quad d_{10}(n) = (\nu_{n,0} + \nu_{n+1,0})/2 - \nu_{n,1}$$

5 point: $dd_{01}(n) = (\nu_{n-1,0} - 4\nu_{n-1,1} + 6\nu_{n,0} - 4\nu_{n,1} + \nu_{n+1,0})/8$

$dd_{10}(n) = -(\nu_{n-1,1} - 4\nu_{n,0} + 6\nu_{n,1} - 4\nu_{n+1,0} + \nu_{n+1,1})/8$



l=0,1 small separations

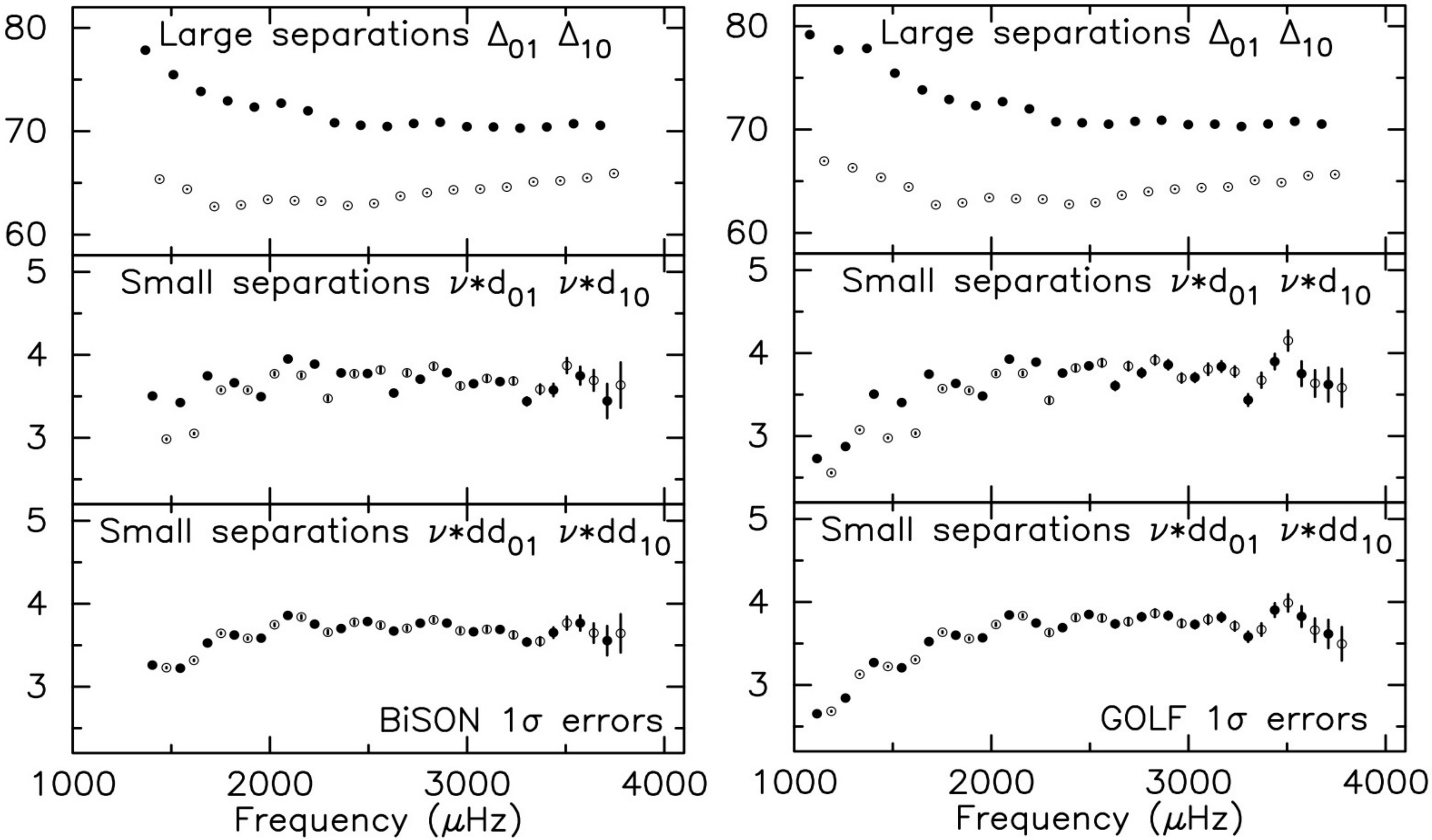
$d_{01}(n)$, $d_{10}(n)$
$dd_{01}(n)$, $dd_{10}(n)$

Both show a mean variation similar to but different from $d_{02}(n)$ giving a diagnostic of the internal structure,

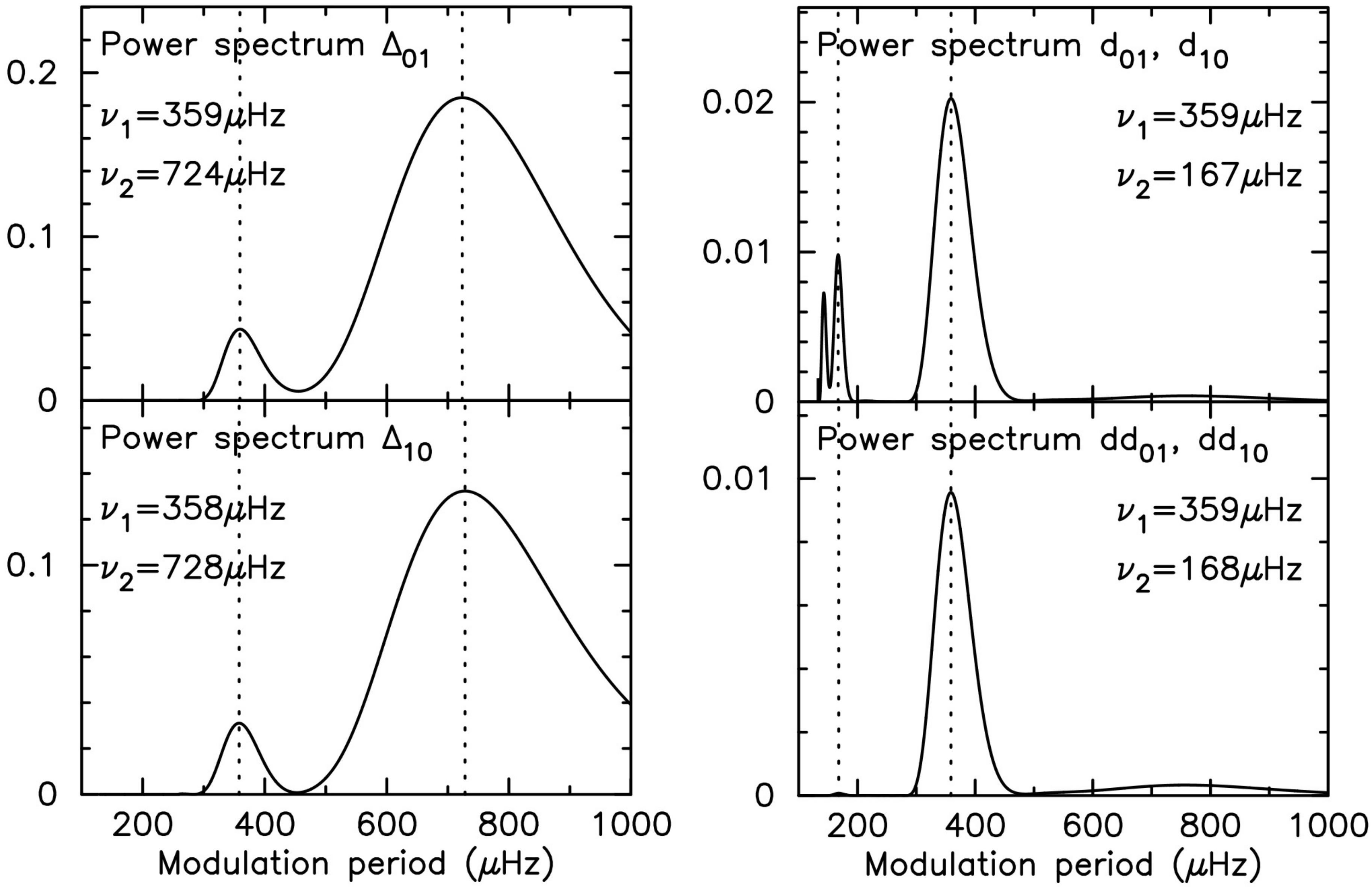
and a periodicity about the mean indicating a region of sharp change

Solar model A

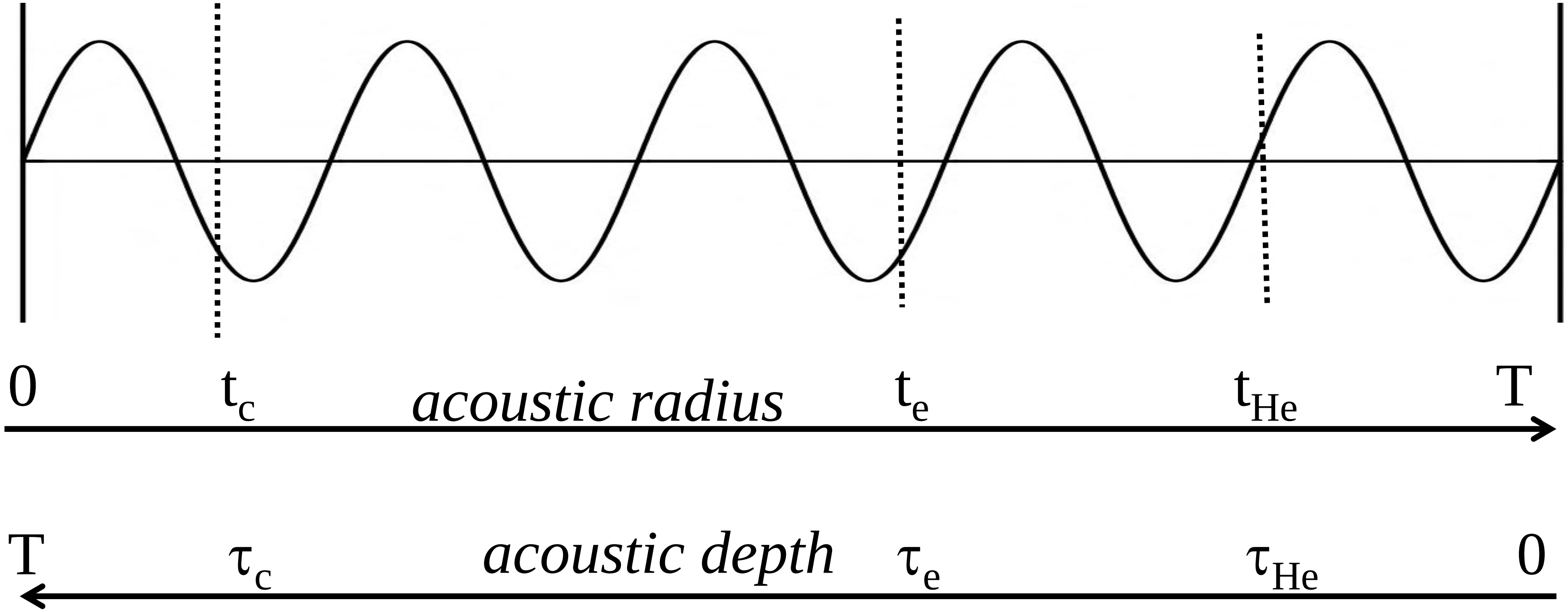
Solar data from BiSON and GOLF



Power Spectra (0,1) separations (Amplitude^2) (μHz^2)



Periodic modulations



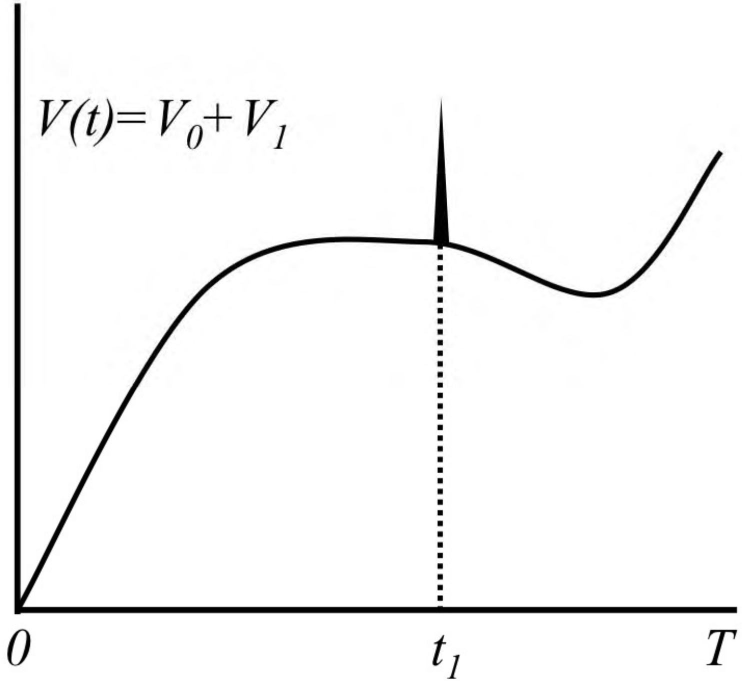
Modulation caused by 'discontinuities' in acoustic variables (c,Γ_1,ρ)

Modulation frequencies $\approx$ *1/(2 distance from boundary)* $1/(2t)$, $1/(2\tau)$

$\Delta = 1/(2T) \approx 136.$ $\quad \tau:\ \nu_{He} \approx 800,\ \nu_e \approx 230.$ $\quad \tau:\ \nu_{He} \approx 165,\ \nu_e \approx 350$

$t + \tau = T \quad \Rightarrow \quad 1/\nu_t + 1/\nu_\tau = 1/\Delta$ *pairs*

Origin of modulation frequencies



$$\frac{d^2\psi}{dt^2} - \frac{\ell(\ell+1)}{t^2}\psi + (\omega^2 - V_\ell)\psi = 0$$

$$\chi = \frac{\omega\psi}{d\psi/dt} = \tan(\omega t + \delta) \qquad t = \int \frac{dr}{c}$$

$$\frac{d\delta}{dt} = -\frac{V_0}{\omega}\sin^2(\omega t + \delta) \quad \rightarrow \quad \delta(\omega, t)$$

$$\Delta\delta = -\int_0^t \frac{V_1}{\omega}\sin^2(\omega t + \delta)\,dt \qquad \Delta\delta(t) = -\frac{[V_1]_-^+}{2\omega}\left[\cos(2\omega t_1 + 2\delta_1) - 1\right] \quad t > t_1$$

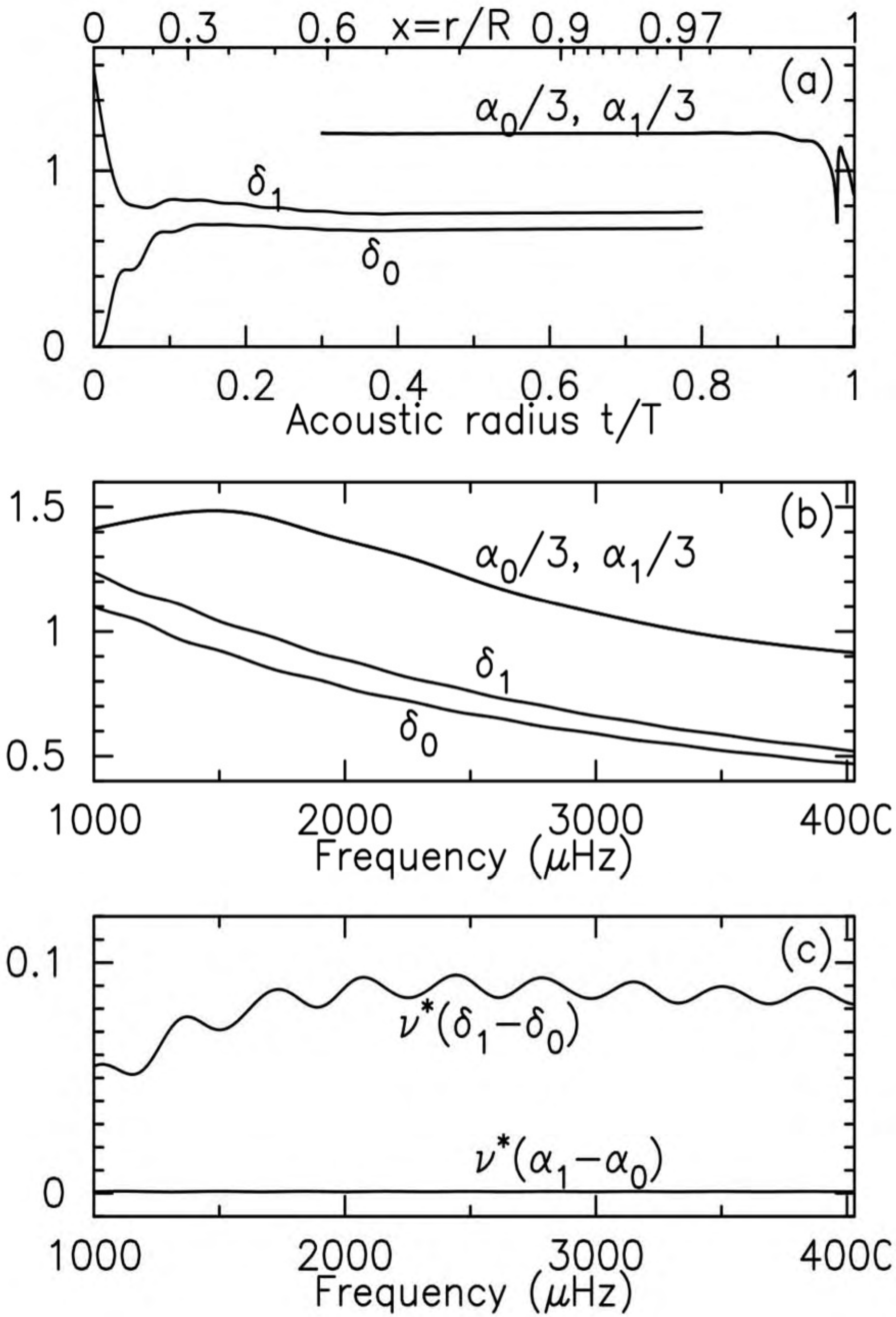
$$\chi = \tan\left[\omega t + \delta(\omega, t) + \Delta\delta(\omega, t)\right] = 0 \quad at \quad t = T$$

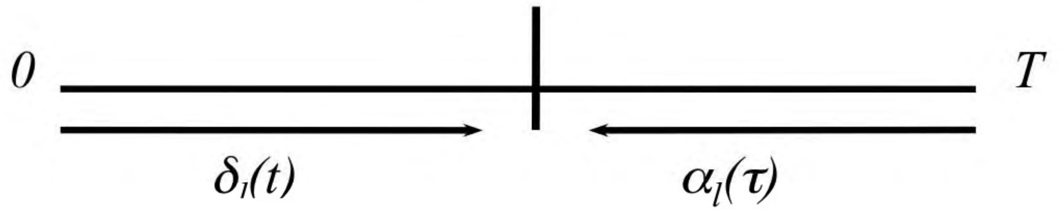
$$\boxed{\omega T = n\pi - \delta(\omega) + \Delta\delta(\omega)}$$

$$\boxed{\Delta\delta(\omega) \propto \cos\left(\frac{\omega}{\omega_1} + 2\delta_1\right) \quad \omega_1 = \frac{1}{2t_1}}$$

$$\delta(\omega) = F(V) = F[\rho(r)]$$

Phase shifts and the eigenfrequency equation





$$\psi_\ell = r p'_\ell/(\rho c)^{1/2}$$

$$\chi_\ell = \frac{2\pi\nu\psi_\ell}{d\psi_\ell/dt} = \tan\left[2\pi\nu t - \frac{\pi}{2}\ell + \delta_\ell(\nu, t)\right]$$

$$\chi_\ell = -\frac{2\pi\nu\psi_\ell}{d\psi_\ell/d\tau} = -\tan\left[2\pi\nu\tau - \alpha_\ell(\nu, \tau)\right]$$

These must be equal at $t_f = T - \tau_f$

$$\boxed{2\pi\nu_{n,\ell}T = \pi\left(n + \frac{\ell}{2}\right) + \alpha_\ell(\nu) - \delta_\ell(\nu),}$$

integer n

Small separations and phase shifts

since $\alpha_0(\nu) = \alpha_1(\nu)$ *and using the eigenfrequency equation*

$$d_{01} = \nu_{n,0} - (\nu_{n-1,1} + \nu_{n,1})/2 \qquad d_{01} = \frac{1}{2\pi T}\left(\delta_1 - \delta_0 - \frac{d^2(\alpha - \delta_1)}{d\nu^2}\frac{\Delta_1^2}{8} + \dots\right)$$

$$d_{10} = (\nu_{n,0} + \nu_{n+1,0})/2 - \nu_{n,1} \qquad d_{10} = \frac{1}{2\pi T}\left(\delta_1 - \delta_0 + \frac{d^2(\alpha - \delta_0)}{d\nu^2}\frac{\Delta_0^2}{8} + \dots\right)$$

to leading order these are determined by δ_1-δ_0, *which is determined by the interior structure*

$$dd_{01}(n) = \frac{1}{2}\, d_{01}(n) + \frac{1}{4}\left(d_{10}(n) + d_{10}(n-1)\right)$$ *cancels out* $d^2\alpha/d\nu^2$

The mean variation of δ_1-δ_0 *determined by the interior structure periodic modulation by boundaries of convective envelope, core ...*

Large separations and phase shifts

$$\Delta_1 = \nu_{n,1} - \nu_{n-1,1} = \frac{1}{2T}\left(1 + \frac{1}{\pi}\left[\frac{d\alpha}{d\nu}\Delta_1 - \frac{d\delta_1}{d\nu}\Delta_1\right] + \dots\right) \approx \frac{1}{2T}$$

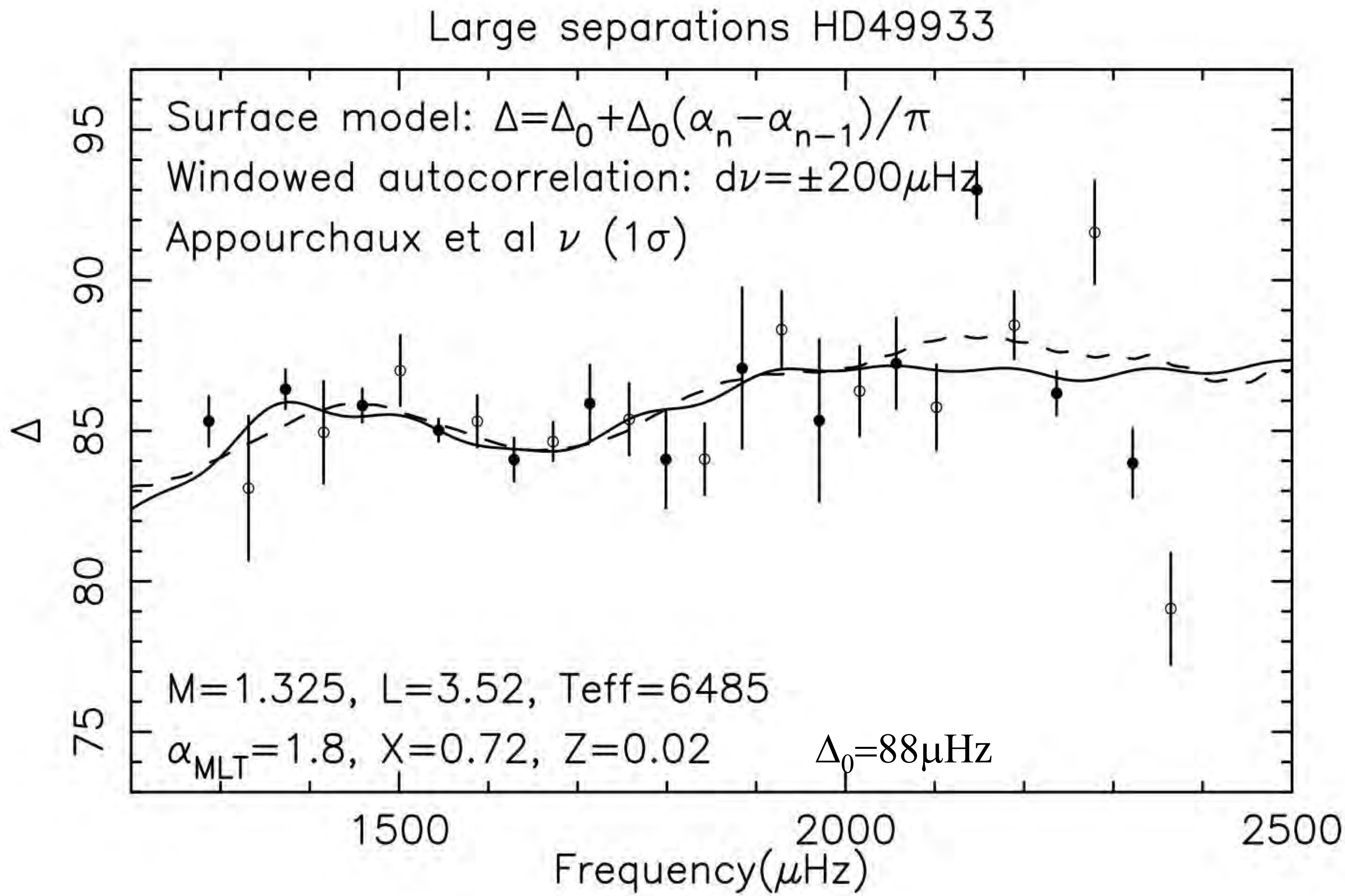
$$\Delta_0 = \nu_{n,0} - \nu_{n-1,0} = \frac{1}{2T}\left(1 + \frac{1}{\pi}\left[\frac{d\alpha}{d\nu}\Delta_0 - \frac{d\delta_0}{d\nu}\Delta_0\right] + \dots\right) \approx \frac{1}{2T}$$

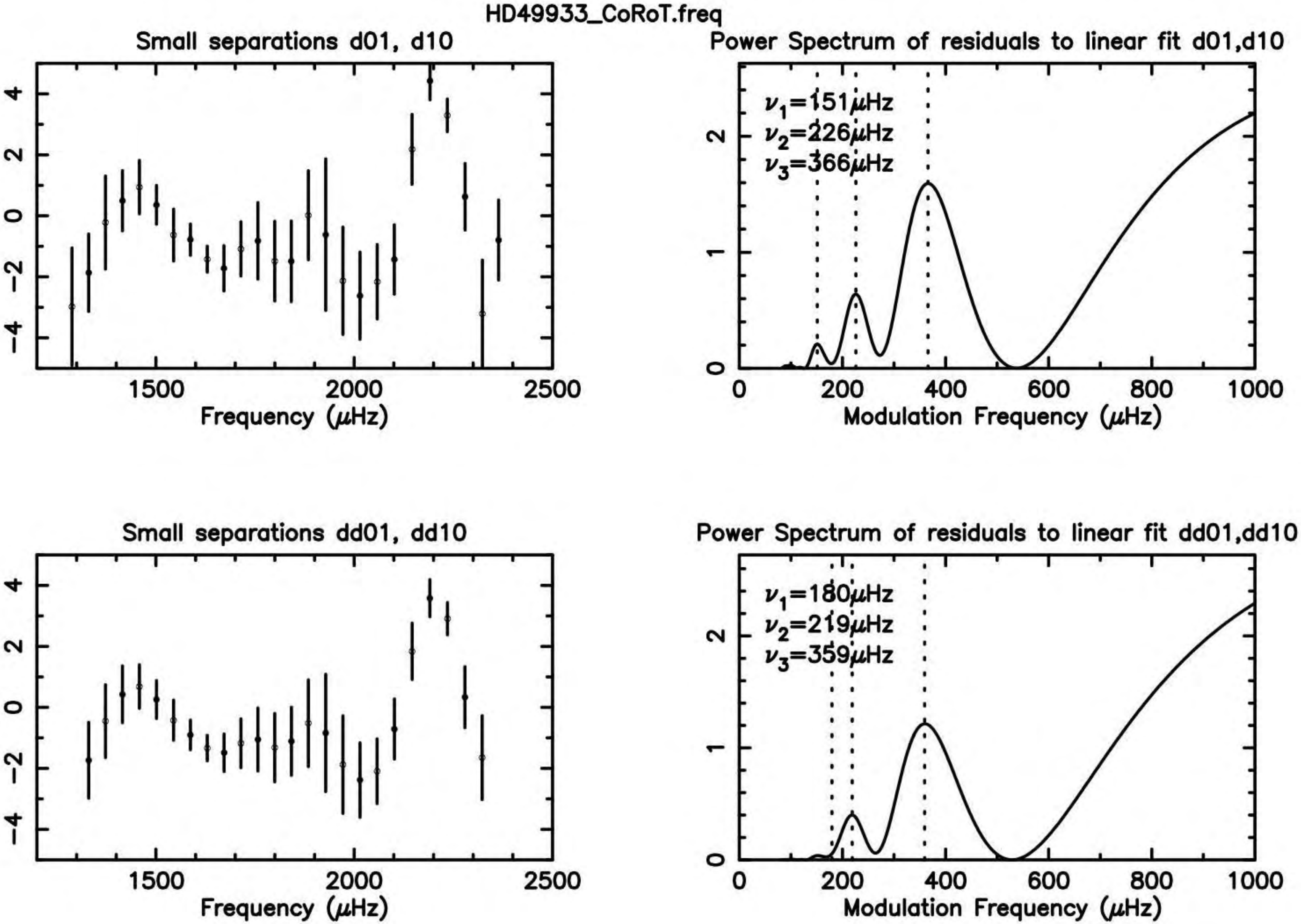
$$\Delta_{01} = \nu_{n,0} - \nu_{n-1,1} = \frac{1}{2T}\left(\frac{1}{2} + \frac{1}{\pi}\left[\delta_1 - \delta_0 + \frac{d\alpha}{d\nu}\Delta_{01} + \frac{d(\delta_0 - \delta_1)}{d\nu}\frac{\Delta_{01}}{2}\right] + \dots\right)$$

$$\Delta_{10} = \nu_{n,1} - \nu_{n,0} = \frac{1}{2T}\left(\frac{1}{2} + \frac{1}{\pi}\left[\delta_0 - \delta_1 + \frac{d\alpha}{d\nu}\Delta_{10} + \frac{d(\delta_1 - \delta_0)}{d\nu}\frac{\Delta_{10}}{2}\right] + \dots\right)$$

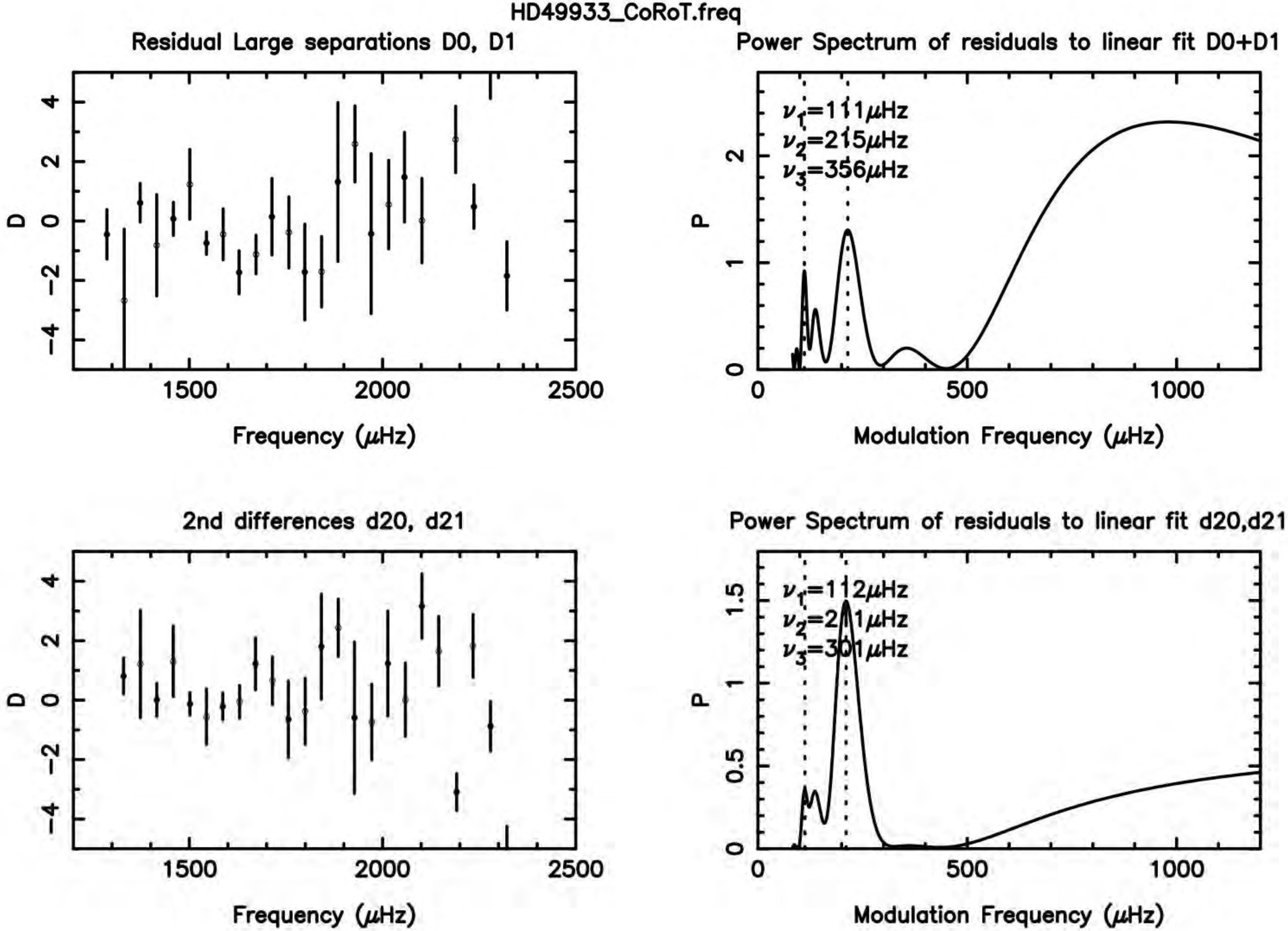
The difference between Δ_{01} *and* Δ_{10} *is primarily due to* δ_1- δ_0 *which is determined by the interior structure*

Can we say anything about CoRoT stars: HD49933 ?





HD49933_CoRoT.freq
Small separations d01, d10
Frequency (μHz)
Power Spectrum of residuals to linear fit d01,d10
ν1=151μHz
ν2=226μHz
ν3=366μHz
Modulation Frequency (μHz)
Small separations dd01, dd10
Frequency (μHz)
Power Spectrum of residuals to linear fit dd01,dd10
ν1=180μHz
ν2=219μHz
ν3=359μHz
Modulation Frequency (μHz)



HD49933_CoRoT.freq
Residual Large separations D0, D1
Power Spectrum of residuals to linear fit D0+D1
ν1=111μHz
ν2=215μHz
ν3=356μHz
2nd differences d20, d21
Power Spectrum of residuals to linear fit d20,d21
ν1=112μHz
ν2=211μHz
ν3=301μHz
D
P
Frequency (μHz)
Modulation Frequency (μHz)

Does it make sense ?

Periods should match in pairs $1/\nu_t + 1/\nu_\tau = 1/\Delta$

Envelope model fitted to large seps gives T~5680s, Δ~88μHz (guide)

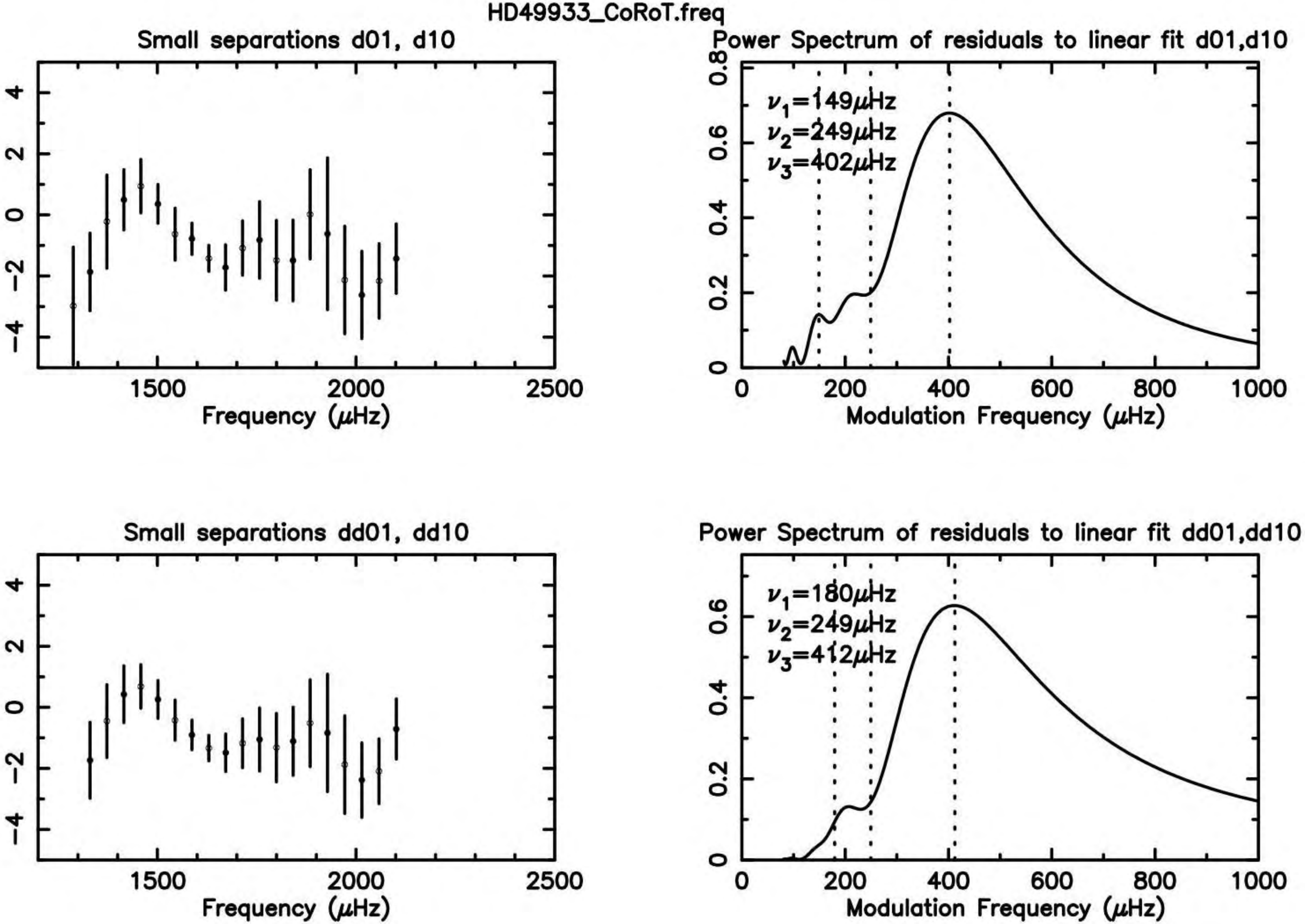
HeII acoustic depth ~ 900secs; ν_{He} ~ 450 - 650 mHz

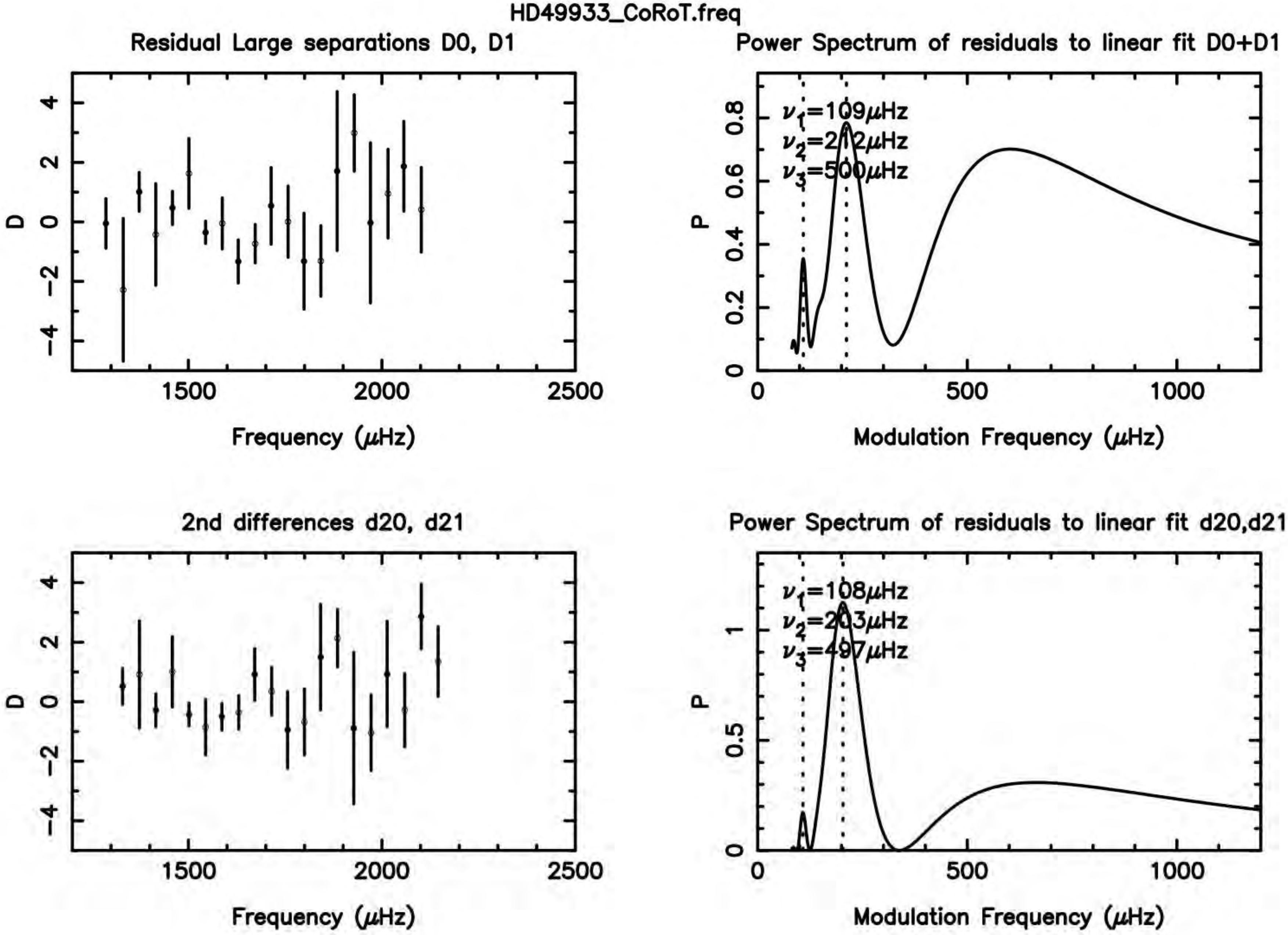
accoustic depth of base Con Zone ~ 2380secs; ν_{CZ} ~ 210μHz

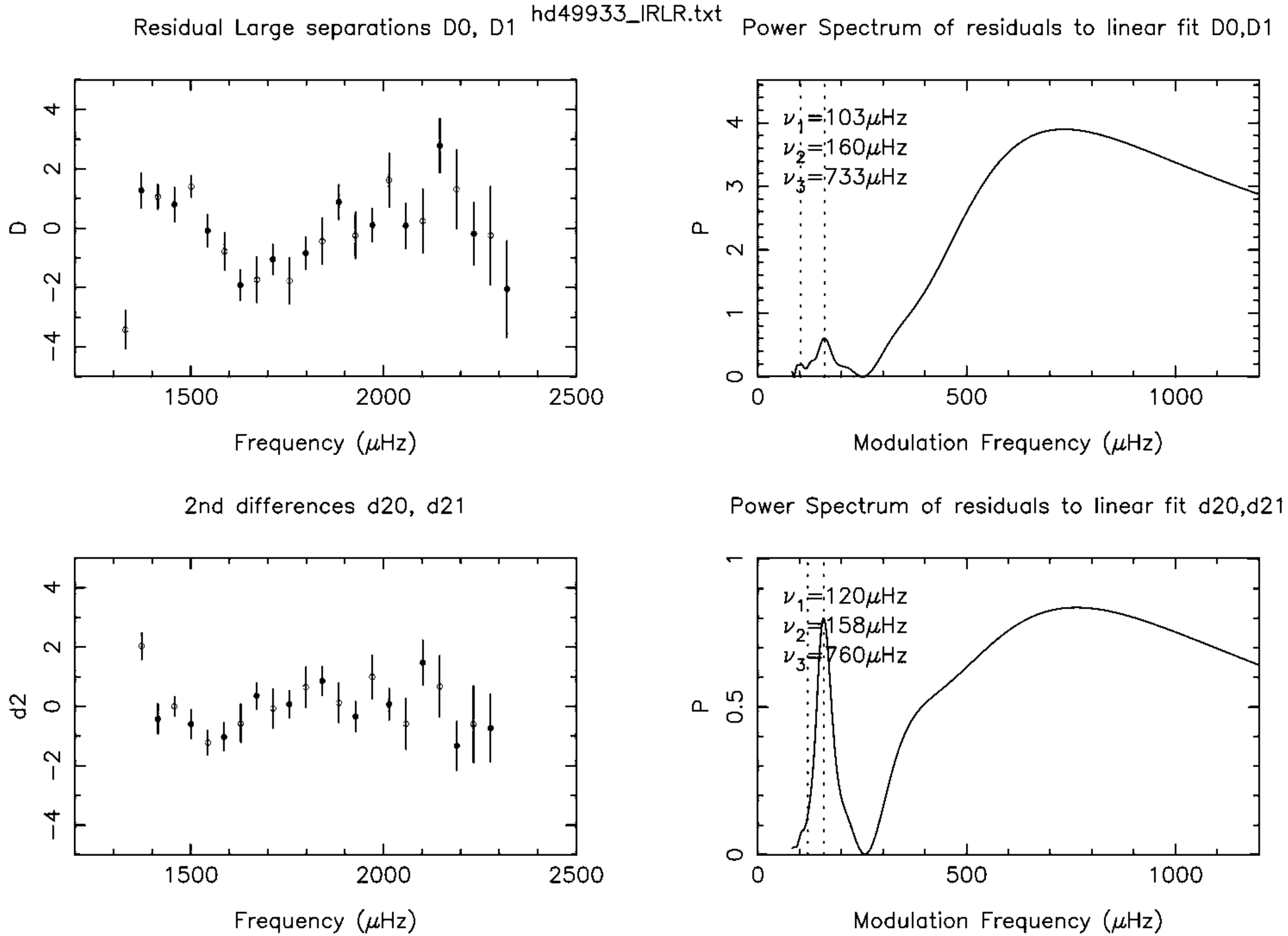
signals at ~ 220μHz and ~ 350μHz should have corresponding signals at ~150 and 120 (Nyquist ~ 86μHz)

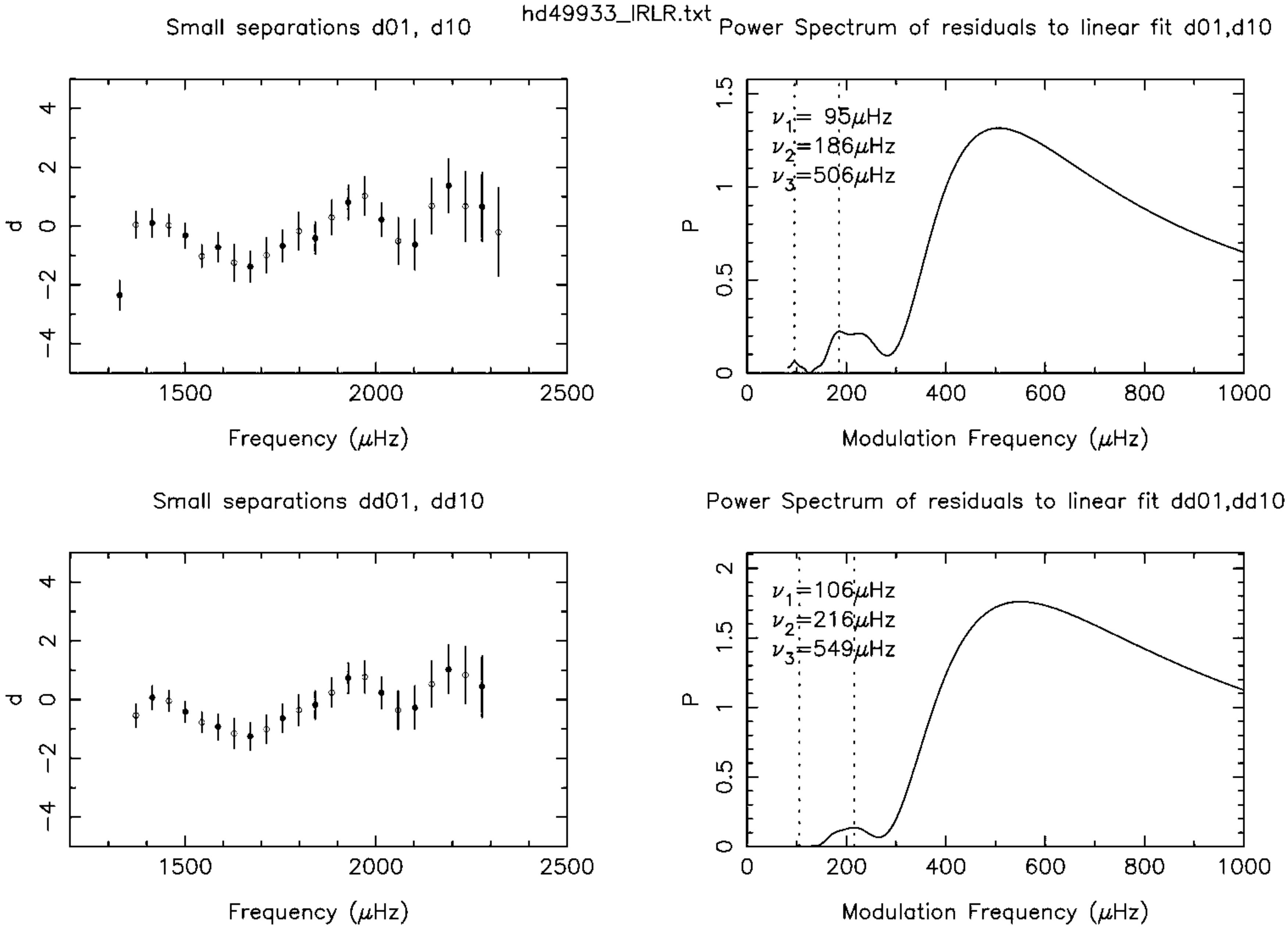
signal ~700 mHz could be HeII zone ; corresponding signal 100μHz

Not convinced frequencies are accurate enough !


HD49933_CoRoT.freq
Small separations d01, d10
Frequency (μHz)
1500
2000
2500
4
2
0
−2
−4
Power Spectrum of residuals to linear fit d01,d10
ν1=149μHz
ν2=249μHz
ν3=402μHz
Modulation Frequency (μHz)
0
200
400
600
800
1000
0.8
0.6
0.4
0.2
0
Small separations dd01, dd10
Frequency (μHz)
Power Spectrum of residuals to linear fit dd01,dd10
ν1=180μHz
ν2=249μHz
ν3=412μHz
Modulation Frequency (μHz)


HD49933_CoRoT.freq
Residual Large separations D0, D1
D
Frequency (μHz)
Power Spectrum of residuals to linear fit D0+D1
ν1=109μHz
ν2=212μHz
ν3=500μHz
P
Modulation Frequency (μHz)
2nd differences d20, d21
D
Frequency (μHz)
Power Spectrum of residuals to linear fit d20,d21
ν1=108μHz
ν2=203μHz
ν3=497μHz
P
Modulation Frequency (μHz)


hd49933_IRLR.txt
Residual Large separations D0, D1
D
Frequency (μHz)
Power Spectrum of residuals to linear fit D0,D1
ν1=103μHz
ν2=160μHz
ν3=733μHz
P
Modulation Frequency (μHz)
2nd differences d20, d21
d2
Frequency (μHz)
Power Spectrum of residuals to linear fit d20,d21
ν1=120μHz
ν2=158μHz
ν3=760μHz
P
Modulation Frequency (μHz)



hd49933_IRLR.txt
Small separations d01, d10
Power Spectrum of residuals to linear fit d01,d10
ν_1= 95μHz
ν_2=186μHz
ν_3=506μHz
Small separations dd01, dd10
Power Spectrum of residuals to linear fit dd01,dd10
ν_1=106μHz
ν_2=216μHz
ν_3=549μHz
d
P
Frequency (μHz)
Modulation Frequency (μHz)

Periods should match in pairs $1/\nu_t + 1/\nu_\tau = 1/\Delta$

158 in 2nd diff: from depth of Czone hence should be signal of ~200 from radius in d01 - there !

If 750 is signal from HeII in D could be signal of ~100 in d01

signal ~ 520 in d01 should be 105 in 2nd diff

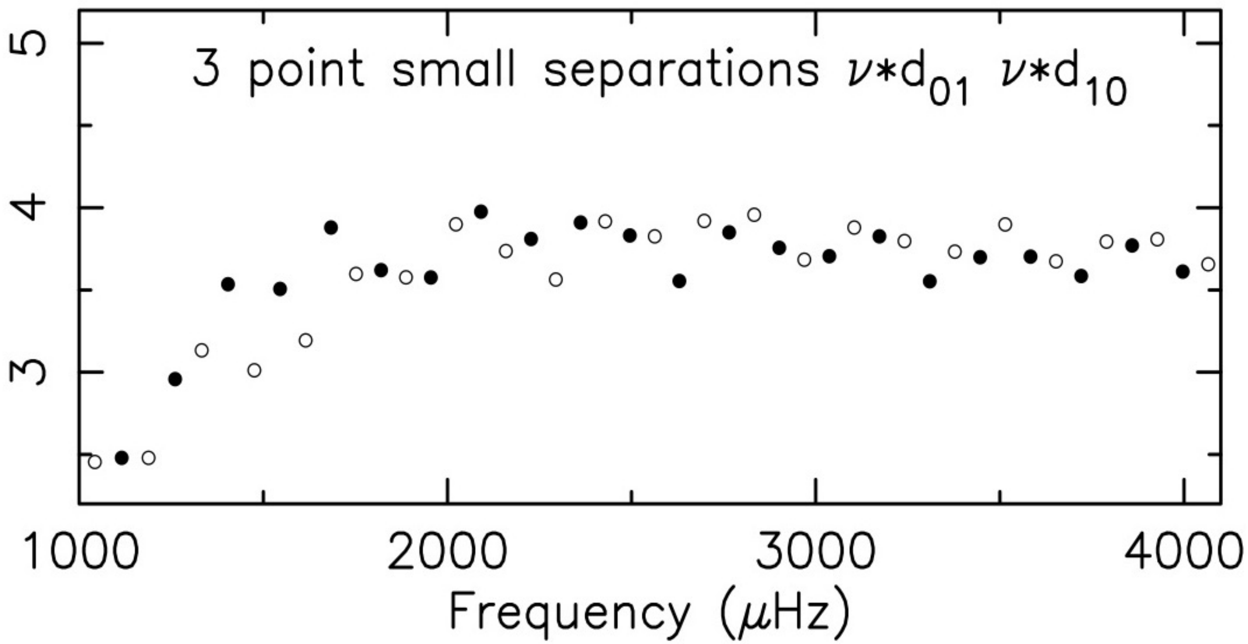
If 700 signal acc depth of HeII zone could be signal 100 in 2nd diff

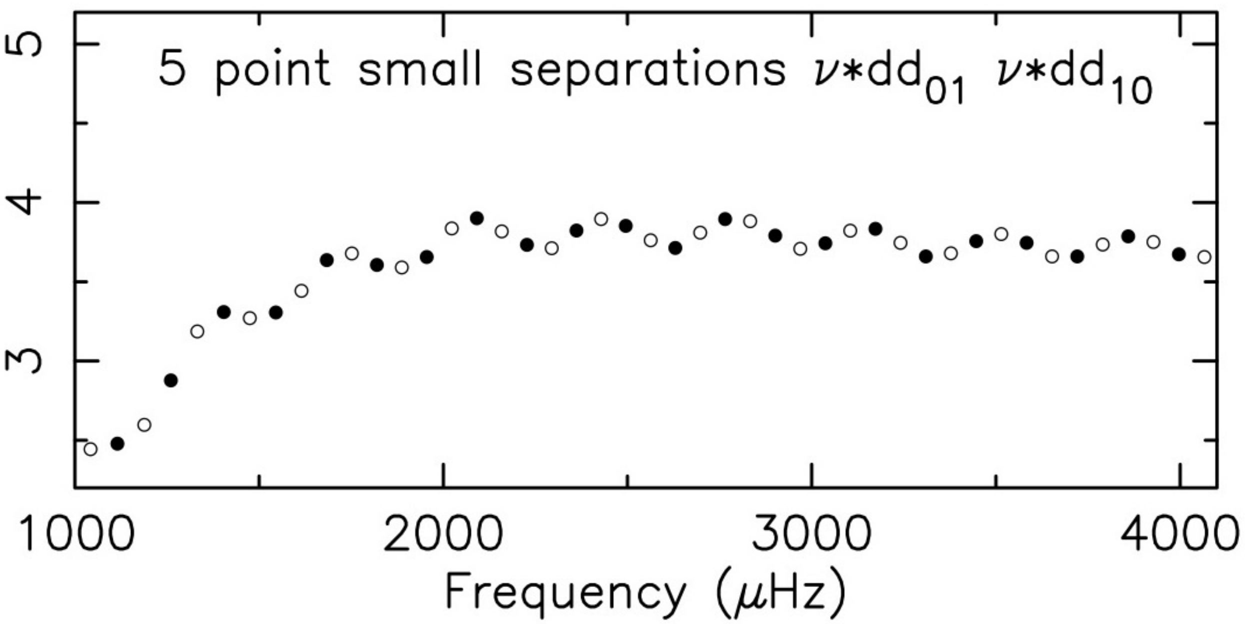
Large seps: 86-88 T = 5680 - 5800

Envelope model fitted to large seps (not nec correct) gives

HeII 450 - 650; BCZ ν ~ 210, T (acc radius) = 5680 (+- 70) secs

should be





l=0,1 separations

$d_{01}(n)=\nu_{n,0}-(\nu_{n-1,1}+\nu_{n,1})/2$

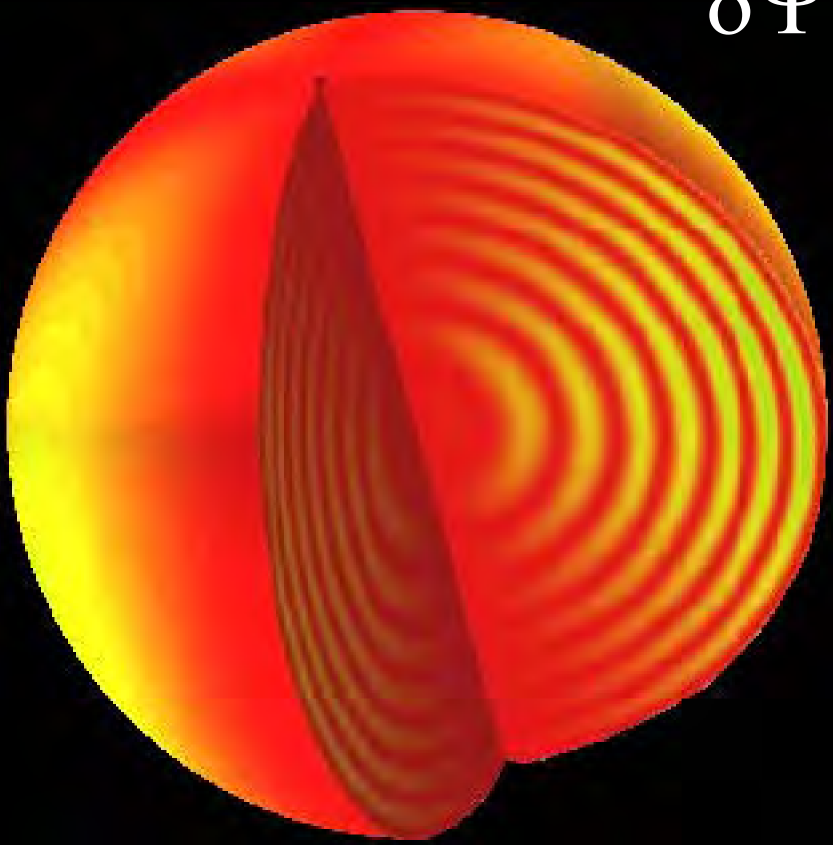
$d_{10}(n)=(\nu_{n,0}+\nu_{n+1,0})/2-\nu_{n,1}$

$dd_{01}(n) = (\nu_{n-1,0} - 4\nu_{n-1,1} + 6\nu_{n,0} - 4\nu_{n,1} + \nu_{n+1,0})/8$

$dd_{10}(n) = -(\nu_{n-1,1} - 4\nu_{n,0} + 6\nu_{n,1} - 4\nu_{n+1,0} + \nu_{n+1,1})/8$

Solar model A

Oscillations of a Spherical Star



n=18, l=2, m=2

$$\delta\Psi(\mathbf{r},t) = \psi_{nlm}(r)\ Y_{lm}(\theta,\phi)\ e^{i2\pi\nu t}$$

$\nu = \nu_{n,l,m}$ $\quad Y_{lm}$ spherical harmonics

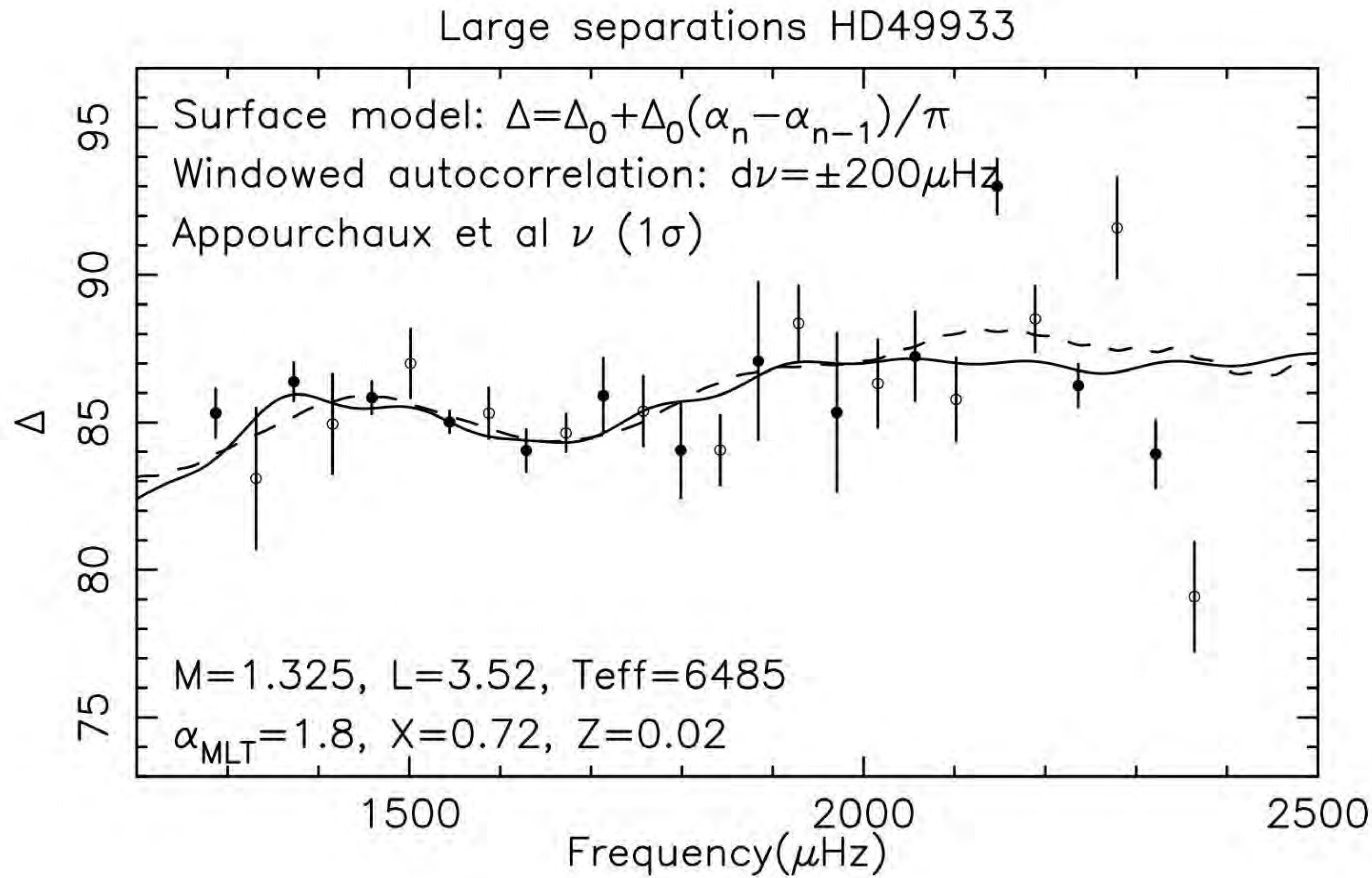
n radial order ≈ no of nodes

l degree, m azimuthal order

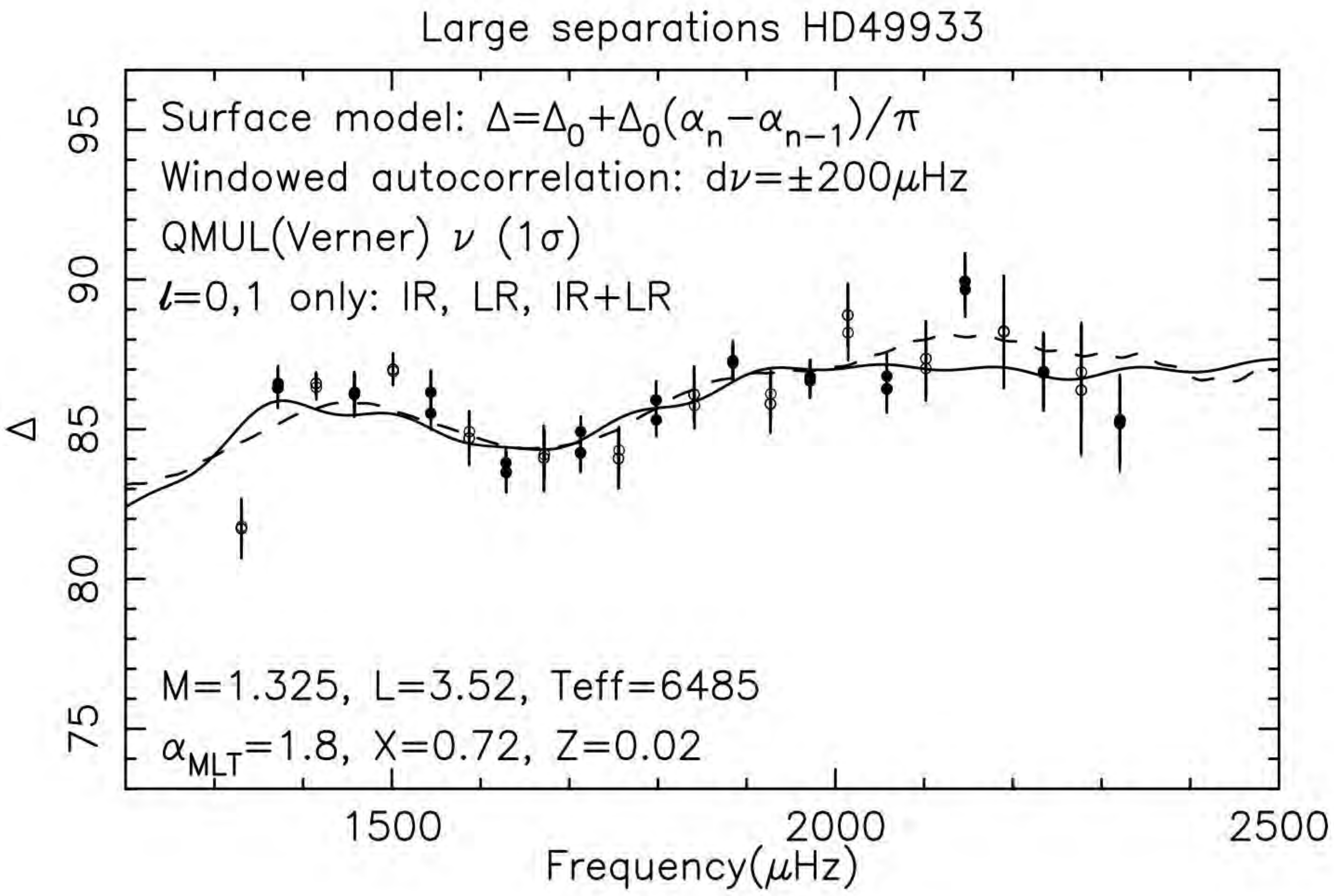
Only modes of low degree observable over integrated disc

m degenerate Ω, $\mathbf{B}$ =0

$$\nu_{n,l,m} = \nu_{n,l,0} \equiv \nu_{n,l}$$



Large separations HD49933
Surface model: $\Delta=\Delta_0+\Delta_0(\alpha_n-\alpha_{n-1})/\pi$
Windowed autocorrelation: $d\nu=\pm200\mu$Hz
Appourchaux et al ν (1σ)
M=1.325, L=3.52, Teff=6485
α_{MLT}=1.8, X=0.72, Z=0.02
Δ
75
80
85
90
95
1500
2000
2500
Frequency(μHz)



Large separations HD49933
Surface model: $\Delta=\Delta_0+\Delta_0(\alpha_n-\alpha_{n-1})/\pi$
Windowed autocorrelation: $d\nu=\pm200\mu$Hz
QMUL(Verner) ν (1σ)
l=0,1 only: IR, LR, IR+LR
M=1.325, L=3.52, Teff=6485
α_{MLT}=1.8, X=0.72, Z=0.02
Δ
75
80
85
90
95
1500
2000
2500
Frequency(μHz)